Developing a Cellular Automaton Model of Urban Growth Incorporating Fuzzy Set Approaches

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Abstract. This is the first part of a two-paper series elaborating the development of a cellular automaton model of urban development using GIS and fuzzy set approaches. Under the paradigm of fuzzy set theory, this paper develops a cellular automaton model of urban development based on an understanding of the logistic trend of urban development processes. The model delimits urban areas as multiple states using a fuzzy membership function and applies transition rules with linguistic variables to represent the non-deterministic nature of urban development controls. By implementing the model in ARC/INFO using the Arc Macro Language (AML) in a GRID environment, experimental scenarios of development of a virtual city under various conditions are presented. Experimental application of the model to an artificial city showed realistic results and demonstrated the model is theoretically feasible and valid. Further work is needed to calibrate the model when applying it to simulate an actual urban development. In the second part of the two-paper series, application of the model in simulating the urban development of Sydney in space and over the last three decades will be demonstrated and discussed.

1. INTRODUCTION

Over the last decade, cellular automata (CA) and their application in urban modelling have been rapidly gaining favour among urban researchers (Batty, 2000, 1998, 1997; Wu and Webster, 2000, 1998; Wu, 1998a, 1998b, 1998c, 1996; Batty, Xie & Sun, 1999; Clarke and Gaydos, 1998; Batty and Xie, 1997; Batty, Couclelis & Eichen, 1997; Clarke, Hoppen & Gaydos, 1997; Couclelis, 1997, 1989, 1985; Wagner, 1997; White and Engelen, 1997, 1994, 1993; White, Engelen & Uljee, 1997; Cecchini, 1996; Itami, 1994). This is because of the ability of cellular automata 'to model and visualise complex spatially distributed processes' (Takeyama and Couclelis, 1997:73). Cellular automata are 'especially appropriate in urban modelling, where the process of urban spread is entirely local in nature and aggregate effects, such as growth booms, are emergent' (Clarke and Gaydos, 1998:700), i.e., their behaviour is generated 'by repetitive application of the rules beyond the initial condition' (Clarke and Gaydos, 1998:700).

Previous studies on urban modelling using cellular automata have addressed various aspects of urban development. However, most of these studies regard urban development as a binary process of non-urban to urban conversion conducted under the paradigm of crisp set theory (Wu and Webster, 2000; Wu, 1998a, 1998b, 1998c, 1996; Clarke and Gaydos, 1998; White and Engelen, 1997, 1994, 1993). The process of urban development resembles a fuzzy process both spatially and temporally. Spatially, there is no sharp boundary between an urban built-up area, urban-rural fringe and non-urban rural land. Temporally, urban development is a continuous process, which follows the general trend of a logistic curve (Herbert and Thomas, 1997; Jakobson and Prakash, 1971). Although Wu (1998b, 1996) developed models of urban development using fuzzy logic control in defining the urban transition rules, he defined the state of cells under the crisp set theory as non-urban or urban. The multiple or fuzzy characteristics of non-urban, partly-urban and urban states in the process of urban development were not addressed.

This paper develops a cellular automaton model of urban development incorporating fuzzy set and fuzzy logic approaches. Urban areas are delimited using a fuzzy membership function and transition rules are applied with linguistic variables to represent the non-deterministic nature of urban development controls. Section 2 demonstrates how development occurs in a cellular space. The impacts of two basic elements of a cellular automaton - the scale of cells and the neighbourhood type - on this development are addressed. In Section 3, the role of fuzzy set theory in defining the state of cells of an urban cellular automaton is discussed, followed by discussions on fuzzy logic control for defining transition rules of an urban cellular automaton. In Section 4, the general trend of urban development over time is used to build a model of urban development incorporating fuzzy logic controlled transition rules of a cellular automaton. By implementing the model in ARC/INFO using the Arc Macro Language (AML) in a GRID environment, experimental development scenarios in a virtual city under various conditions are presented. Through this experimentation the validity of the model is verified. Application of the model to a real city - the metropolitan region of Sydney - will be presented in the second part of the two-paper series.

2. AN URBAN CELLULAR AUTOMATON AND ITS DEVELOPMENT

A cellular automaton, or CA for short, is a discrete dynamic system in which space is divided into regular spatial units called cells and time progresses in discrete steps. Each cell in the system has one of a finite number of states. The state of each cell is updated according to local rules, i.e., the state of a cell at a given time depends on its own state and the states of its nearby neighbours at the previous time step (Wolfram, 1984). In this section, a cellular automaton based city is proposed, which consists of $n \times n$ spatial cells. The state of a cell represents an area subject to specific urban development processes. Section 2.1 presents a generic principle of development of a cellular automaton, followed in Section 2.2 by discussions on the effects of the cell scale and the type/size of neighbourhood on the behaviour of the cellular automaton. Examples of implementing various transition rules in generating greater reality of development are presented in Section 2.3.

2.1 Generic principle of development for a cellular automaton

Let $S^t_{x_{ij}}$ be the state of a cell x_{ij} at the location i, j at time t. $S^t_{x_{ij}}$ belongs to a finite number of states of cells in the cellular space. Let $S^{t+1}_{x_{ij}}$ be the state of the cell at time t+1. Then,

$$S^{t+1}_{x_{ij}} = f(S^t_{x_{ij}} , S^t_{\Omega_{x_{ij}}}) \qquad (2.1)$$

where $\Omega_{x_{ij}}$ represents a set of cells at the neighbourhood of cell x_{ij}, $S^t_{\Omega_{x_{ij}}}$ is a set of states of cells $\Omega_{x_{ij}}$ at time t, and f is a function representing a set of transition rules.

Consider the cell itself as a member of its neighbourhood, then Equation 2.1 can be written as:

$$S^{t+1}_{x_{ij}} = f(S^t_{\Omega_{x_{ij}}}) \qquad (2.2)$$

Equation 2.2 can be expressed in a verbal form, which illustrates a generic principle of development of a cellular automaton, i.e.

IF something happens in the neighbourhood of a cell,
THEN something else will happen to the cell at the following time step.

A cellular automaton model usually consists of a set of 'IF-THEN' statements which imply specific transition rules. For instance, the famous model '*Game of Life'* (Gardner, 1972) can be expressed as three 'IF-THEN' statements:

IF there are two or three live cells in the Moore Neighbourhood of a live cell,
THEN the cell stays alive in the next generation;
IF there are less than two or more than three live cells in the Moore Neighbourhood of a live cell,
THEN the live cell dies in the next generation;
IF there are exactly three live cells in the Moore Neighbourhood of a dead cell,
THEN the dead cell becomes alive in the next generation.

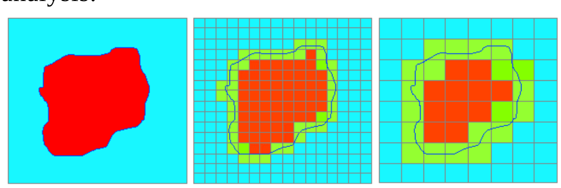
In defining the 'IF-THEN' statements, the four basic elements of cellular automaton - the cells, the states, the neighbourhood and the transition rules - need to be specified. For an urban system, the cells can be a locale developing from non-urban to urban; the states, types of land such as urban or non-urban, or any specific land use types; the neighbourhood, regions where development might take place; and the transition rules, rules that affect transition of cells from one state to another, implying the process of development in the locale.

Due to the generic principle of development, cellular automaton models 'may serve as a framework for modelling complex natural phenomena in a way that is conceptually clearer, more accurate, and more complex than conventional mathematical systems' (Itami, 1994:30).

2.2 The scale of cells and the neighbourhood

Scale of cells

For models of spatial phenomena, scenarios resulting from the tessellation of space at different scales vary (Figure 1). This is commonly regarded as a modifiable area unit problem (MAUP). Openshaw (1984) provides a comprehensive review on the early research on the modifiable area unit problem. Fotheringham and Wong (1991) explain that the modifiable area unit problem is essentially unpredictable in its intensity and effects in multivariate statistical analysis and is therefore a much greater problem than in univariate or bivariate analysis.



(a) (b) (c)

Figure 1: The modifiable area unit problem in spatial modelling.

Red cells: urban; green cells: fringe; blue cells: non-urban.

a) an urban area sited in a regional context; b) the area tessellated into cells at a small scale; c) the area tessellated into cells at a large scale. This figure shows that different patterns of urban, fringe or non-urban can be achieved by tessellating the urban area at different scales.

In the application of cellular automaton in urban development modelling, the scale of cells of a cellular automaton varies significantly. For instance, Wu (1998b, 1998c, 1996) used both 28.5-metre and

200-metre cells to model an area of 224 square kilometres. White and Engelen (1993) constructed a cellular automaton model with 500-metre grid size to simulate the urban land use patterns in a set of US cities; they also modified the model to a 'high-resolution' with 250-metre grid size to simulate urban land use dynamics in the city of Cincinnati, Ohio State (White, Engelen & Uljee, 1997:323). Clarke and colleagues used a basic grid of 300-metre cells for the San Francisco Bay area (Clarke, Hoppen & Gaydos, 1997). However, while applying the model to the Washington/Baltimore region, calibrations were undertaken at resolutions of 210, 420, 840 and 1680 metres respectively (Clarke and Gaydos, 1998). Their results show that although not all rules or factors are sensitive to the change of the cell scale, the scale of cells does have impacts on results of the simulation, especially in relation to some factors such as road and slope factors. They suggested a hierarchical approach in calibrating the model by 'first using coarse data to investigate the scaling nature of each parameter in a different city setting, then scaling up once the best data ranges are found' (Clarke and Gaydos, 1998:710).

In this paper, the model of urban development was constructed in a scale-independent mode. However, when applying the model to simulate the process of urban development of a specific region, this model will be calibrated with data at different cell scales. Effects of the scale of cells on the simulation results can be evaluated and the model be fitted with the best data range.

Neighbourhood size/type

According to the theory of cellular automaton, the global behaviour of a self-organizing system is governed by locally defined transition rules. For an urban system, a fundamental question is to what extent urban development is a locally specified process (Wu, 1996). Some factors, such as slope and height of land affect urban development in a small area base; others such as urban planning and the transportation networks are global controls over the whole area. Moreover, developments in information technology and telecommunications have had fundamental consequences for the patterns and processes of urban change throughout the world (Herbert and Thomas, 1997). These factors affect urban development in a universal way. In practice, both small and large neighbourhood sizes have been applied to models of urban development. The former being a nine-cell Moore Neighbourhood as was applied in Clarke and Gaydos (1998), Wu (1998a, 1998b, 1998c, 1996) and Clarke, Hoppen and Gaydos (1997), and the latter being 113 cells surrounding a cell in question, as was applied in White and Engelen (1994, 1993). No particular validation on the size of neighbourhood in cellular automata based urban models has been explored. However, most applications of cellular automaton models in urban research apply a larger neighbourhood size than applications in natural sciences (Batty and Xie, 1994). This is probably because of the difficulty in justifying transition rules in behavioural terms (Wu, 1996) and the existence of distance-decay effects of the neighbouring cells to the central cell in question (Wu, 1996; White and Engelen, 1994, 1993).

Regardless of the size of the neighbourhood, the type of neighbourhood also has significant impacts on the behaviour of a cellular automaton. Li and Yeh (2000) shows that the use of a rectangular neighbourhood such as the Moore Neighbourhood might produce significant distortions between cells at different directions from a circular object (Li and Yeh, 2000).

In fact, the application of a rectangular neighbourhood in a cellular automaton model can produce distortion on an object of any shape. This is due to the existence of distance-decay effect of the neighbouring cells on the central cell in question (Figure 2). The distortion is especially significant when a large neighbourhood size applies, which can be eliminated by applying a circular neighbourhood.

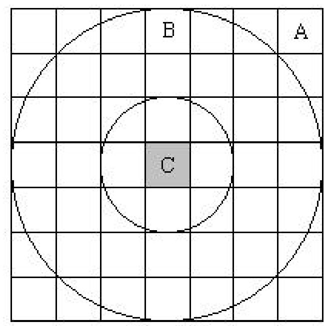


Figure 2: Distortion produced by a rectangular neighbourhood
C is the processing cell. For a rectangular neighbourhood of 7 by 7 cells, the effect of cell A on the processing cell (C) differs from that of cell B, although both A and B are in the same row of the neighbourhood. This is because the distances from the centres of A or B to the processing cell C are different.

In this paper, a circular neighbourhood was defined by specifying a radius in cell(s) from the centre of the processing cell. Any cell centre encompassed by the circle was included as a neighbour of the processing cell. Three different radiuses representing a small, a medium and a large neighbourhood size were tested (Figure 3). The radius of the small neighbourhood size was set to one-and-a-half cells with the medium two-and-a-half cells and large three-and-a-half cells respectively. In comparison with the simulation results, impacts of these neighbourhood sizes on the model's output were assessed with the model.

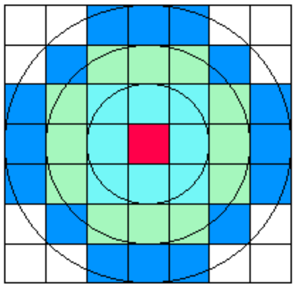


Figure 3: Three sizes of neighbourhood
Small neighbourhood: Turquoise cells;
Medium neighbourhood: Turquoise and green cells; Large neighbourhood: Turquoise, green and blue cells.

2.3 Generating greater reality with transition rules

Development of a cellular automaton is controlled by a number of transition rules. These transition rules are usually expressed as a set of 'IF-THEN' statements, which are intrinsically simple. However, these simple rules can generate complex patterns of development. Consider a locale with 100 by 100 cells, each representing a square area of 2500 square metres. Only five cells at the centre of the area were developed into urban state (Figure 4, t = 0). Assume that the geographical condition of all cells in this area is identical and the only force driving the development of cells is the number of developed cells in the neighbourhood of a cell in question, implying the growth of new urban cells from the urbanized cells. In an 8-cell neighbourhood (the Moore Neighbourhood), the transition of the state of cells is governed by the following rule:

IF there are three or more developed cells in the Moore Neighbourhood of a cell,
THEN the cell is developed.

The model generates a scenario of urban development as shown in Figure 4.

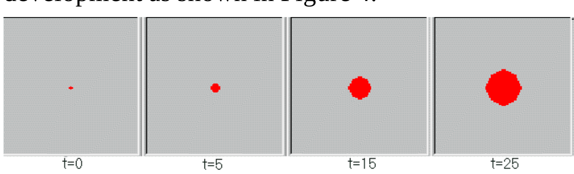


Figure 4: A CA generated urban development in a plain area (Moore Neighbourhood)
Red: urban; Grey: non-urban; t: time step.

In a real situation the geographical condition of an area can never be identical; therefore, there are other rules controlling the transition of the state of cells. In this case, more IF-THEN statements need to be added to represent different transition rules. For instance, in the above example, if there is a road from the centre to the northeast part of the area, more development can occur along the direction of the road. Therefore, another IF-THEN statement needs to be added to the first rule to implement the road attracted development.

IF there are one to two developed cells in the Moore Neighbourhood of a cell, and there is a road running through this cell,
THEN the cell is developed.

With these two rules, the scenario of urban development in this area changes, as shown in Figure 5.

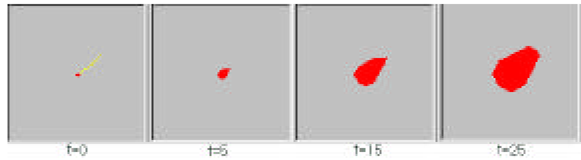


Figure 5: A CA generated urban development with the effect of a road (Moore Neighbourhood)
Red: urban; Grey: non-urban; Yellow: road;
t: time step.

In addition to the existence of a road, there may have a river across the centre of the city, and a cell cannot be developed if the river runs across it. In this case, another rule can be added to the cellular automaton to illustrate the constraint of this factor, which can be expressed as follows:

IF a cell crosses a river,
THEN no development will happen to that cell.

Again, with the implementation of this new rule, the pattern of development in this area changes, as shown in Figure 6.

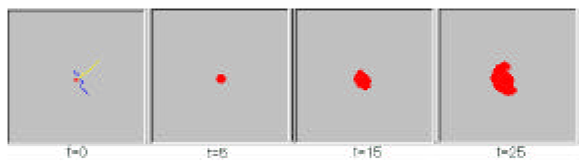


Figure 6: A CA generated urban development with the effects of both a road and a river (Moore Neighbourhood)
Red: urban; Grey: non-urban; Yellow: road;
Blue: river; t: time step.

Although the above example is a very simplified one, it provides a general idea of how locally made transition rules can be implemented in a cellular automaton model, and how these rules can be applied to simulate the complex behaviour of systems in a cellular space. However, rules implemented in these examples were all deterministic, which were based on classical set theory. Recent development shows that the transition rules of a cellular automaton are not restricted to only deterministic forms. More flexibility in defining these rules such as the application of probability concept and fuzzy logic has been tested (Wu, 1996; White and Engelen, 1993). As urban development is the result of both physical constraints and human decision-making behaviours which are characterized with uncertainty and fuzziness, applications of fuzzy set theory and fuzzy logic control seem attractive in defining the rules controlling urban development.

3. URBAN DEVELOPMENT AND FUZZY SETS

Urban development is a process of physical concentration of people and buildings (Herbert and Thomas, 1997). This is a continuous process in space and over time which resembles a fuzzy process in both its definition of urban areas and the factors controlling the development. The fuzzy characteristics of urban development imply the applicability of the fuzzy set approach in modelling the process of this development.

3.1 Urban state and fuzzy set

Representation of geographical boundaries

Traditionally, it is very common to use thematic maps for the representation of geographical phenomenon (Woodcock and Gopal, 2000; Wang and Hall, 1996). In thematic maps, the use of categories has followed classical set theory, where each location is assumed to belong to a single category; the boundaries between different categories are represented as sharp lines (Woodcock and Gopal, 2000; Burrough, 1986). This representation might be accurate when dealing with cadastral, census or administrative boundaries that are 'sharply defined' (Wang and Hall, 1996:574). However, it is not accurate in representing boundaries of land features with continuously changing properties, such as soil quality, land cover or population densities as such boundaries are rarely sharp or crisp. In this case, the representation of geographical boundaries based on crisp set theory may lead to misunderstanding of the information represented (Wang and Hall, 1996).

Like many dynamic processes of geographical phenomena, urban development is a continuous process both spatially and temporally. Spatially, an urban area is normally defined as an area with high population density and the dominance of non-agricultural land. This is quite a fuzzy definition and there is a variety of differences in determining how high the population density should be before an area can be regarded as urban. Moreover, all cities are surrounded by rural or natural land and there are no sharp boundaries between an urban built-up area and its non-urban hinterland. Between the well-recognized urban land use and the area devoted to agriculture, there exists 'a zone of transition in land use, social and demographic characteristics, lying between a) the continuously built-up area and suburban areas of the central city, and b) the rural hinterland, characterized by the almost complete absence of non-farm dwellings, occupations and land use' (Pryor, 1968:206). This 'zone of transition' is a place where both urban and non-urban features occur, which has been broadly termed as 'fringe' or 'rural-urban fringe' (Bryant, Russwurm & McLellan, 1982; Pryor, 1968). This fringe area has become the most vigorous part of development in the rural-urban continuum and has attracted much attention in research. However, due to its complexity in both spatial and socio-economic features, its constant changing characteristics and the diversity of the extent of development in this area, different concepts or terminology have been applied in addressing such research, such as 'suburb', 'fringe', 'urban fringe', 'rural fringe', 'inner fringe', 'urban shadow zone', 'exurban zone' or even 'rurban fringe' (Bryant, Russwurm & McLellan, 1982; Carter, 1976; Kurtz and Eicher, 1958-1959; Wehrwein, 1942). Obviously, this terminology is imprecise making the study of the spatial structure of urban systems very complex and less comparable (Kurtz and Eicher, 1958-59).

Temporally, if an area has been developed from one state (non-urban) to another (urban) at a certain period of time, development has actually taken place continuously within this period. Therefore, it is difficult and inaccurate to define a sharp time spot when the actual change of state occurred.

Therefore, the basic problems referring to the representation of geographical boundaries are twofold:

- How to represent spatial changes to continuous geographical boundary?
- How to represent temporal changes to continuous geographical boundary?

The development of fuzzy set theory and its application for representing geographical phenomena have provided a solution in dealing with such problems. Unlike crisp set theory where a location in a landscape either belongs to a map category exclusively or it does not belong to it at all, fuzzy sets allow partial belonging represented by a grade of membership in this fuzzy set. The following sections present the terminology of fuzzy set theory and its application in delimiting urban areas.

Fuzzy set theory

Fuzzy set theory was developed in the 1960s by Zadeh (1971, 1965, 1962). This theory was proposed to extend crisp set theory in order to deal with continuous classifications. A set is fuzzy if an element can belong partly to it, rather than having to belong completely or not at all. Therefore, fuzzy set theory begins with the assignment of membership grades to elements which are not restricted to 0 (non-membership) or 1 (full-membership), but which may lie somewhere in the interval from 0 to 1. Mathematically, a fuzzy set can be expressed as follows:

Let X be a collection of objects, whose generic element is denoted as x. Thus X = {x}. A fuzzy set A in X is a set of ordered pairs,

$$A = \{(x,\ m_A(x)) \mid x \in X\} \tag{3.1}$$

where $m_A(x)$ represents the grade of membership of x in A, which associates with each x a real number in [0,1].

Consider a city in a regional context. Within this region, some areas have been fully developed as urban built-up areas, such as the central business district (CBD), the highly populated residential areas or the concentrated industrial zones. Some areas remain in a non-urban state, such as the surrounding open or agricultural land or the regional recreational land. These areas are identified as non-urban areas. Except for the extreme categories, there are areas that have been developed to some extent, such as areas with low to medium population density or with both agriculture and industries. These areas can be categorized as partially developed areas. The temporal dimension of this development is similar to that of its spatial dimension. If an area has been developed from non-urban to urban built-up area within a certain period, development has been a continuous process. Therefore, within this time span the area could have been partially developed to some extent. To illustrate the extent of development of this city in space and over time, a fuzzy concept of 'urban' or 'non-urban' can be defined.

Let X be a collection of cells representing an area in a regional context. x_{ij} is a generic form of a cell in X. An urban fuzzy set S_{urban} can be defined as a set of ordered pairs,

$$S_{urban} = \{(x_{ij},\ m_{S_{urban}}(x_{ij}))|\ x_{ij} \in \mathrm{X}\} \quad (3.2)$$

where the $m_{S_{urban}}(x_{ij})$ is a membership function of the cell x_{ij} in the fuzzy set S_{urban}, the value of which represents the state a cell undergoing an urban development process. Similarly, a non-urban fuzzy set can also be defined as:

$$S_{non-urban} = \{(x_{ij},\ m_{S_{non-urban}}(x_{ij}))|\ x_{ij} \in \mathrm{X}\} \quad (3.3)$$

where the $m_{S_{non-urban}}(x_{ij})$ is a membership function of the cell x_{ij} in the fuzzy set $S_{non-urban}$.

The membership function determines how and to what degree a cell belongs to the set. It depends on the extent a cell is being developed on the urban growth process. The closer the grade of membership is to 1, the higher the degree of membership of the cell in that fuzzy set. An example of the fuzzy membership functions $m_{S_{urban}}(x_{ij})$ and $m_{S_{non-urban}}(x_{ij})$ is presented in Figure 7.

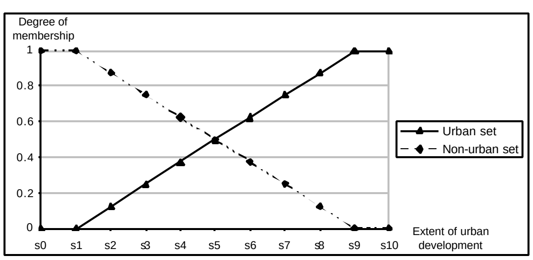


Figure 7: Example membership functions of urban and non-urban fuzzy sets
Stages s0 to s10 represent different extents of urban development, from none to fully developed.

In this example, if a cell has a membership grade of 0.8 in the urban fuzzy set, it has been developed to a higher extent than a cell with a membership grade of 0.3. Conversely, a cell with a membership grade of 0.8 in the non-urban fuzzy set has been less developed than a cell with a membership grade of 0.4 in that set. With this terminology, the boundary between non-urban and urban areas can be understood not as a sharp line, but a region with continuous change on the scale of membership.

Obviously, the membership function is a crucial component of a fuzzy set. Different membership functions represent different fuzzy sets, even though they may have similar context. Figure 8 illustrates three membership functions, one using a linear function, one using an exponential function and one using a logarithmic function. Each function represents a different fuzzy set although they all have similar context which is 'a class of cells been developed'. To determine whether a particular function is suitable for a set or not depends on the context of a particular application.

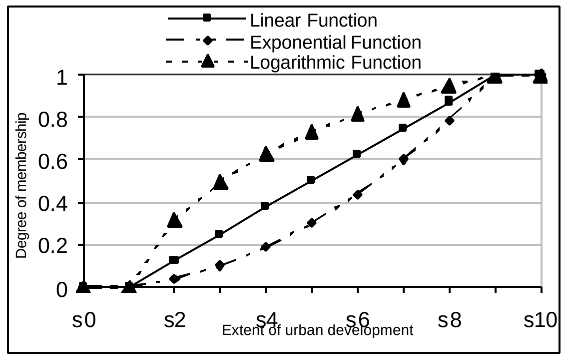


Figure 8: Variations of fuzzy membership functions
This figure illustrates three different membership functions. s0 to s10 in the X-axis represent different extents of urban development. They define three different fuzzy sets although they have similar content, i.e., before s1, the area is undeveloped and after s9 the area is fully developed. Development has taken place in areas with an urban extent between s1 and s9. However, their membership grades vary, which are determined by their membership functions.

3.2 Defining urban states with fuzzy set approach

Consider the fuzzy set 'urban' defined in Equation 3.2:

$$S_{urban} = \{(x_{ij},\ m_{S_{urban}}(x_{ij}))|\ x_{ij} \in \mathrm{X}\}$$

The value of its membership function $m_{S_{urban}}(x_{ij})$ ranges from 0 to 1, which represents the state a cell undergoing an urban development process. For instance, if a cell has a membership grade of 0, this cell has not been developed, i.e., it is in a non-urban state; if a cell has a membership grade of 1, it has been fully developed as an urban area, i.e., it is in an urban state. Cells with a membership grade between 0 and 1 have been developed to some extent, although not fully developed. To define a mathematical formula for the fuzzy membership function, a measurable representation of the extent of urban development of cell x_{ij} in the urban fuzzy set S_{urban} needs to be defined.

Measurement of urban development

A number of approaches have been applied to measure the extent of urban development. These include the use of detailed rules of the size and density of population for the definition of urban areas, the use of pure population densities and the use of the remote sensing approach. As cities are physical agglomerations of population and housing, it is thought that there exists a threshold along the population-size continuum of settlements at which a village becomes a town, although this threshold varies significantly in space and over time (Herbert and Thomas, 1997). Various rules on the size and density of population have been applied for the definition of urban areas. For instance, in several Scandinavian countries, including Denmark and Sweden, any settlement which has more than 200 inhabitants is classified as urban in the national census (Herbert and Thomas, 1997); in the United States, the ‘urban areas’ comprise places with 2,500 or more inhabitants and some special types of areas having a density of 1,500 inhabitants per square mile (about 580 inhabitants per square kilometre) (USBC, 1960). In Canada, an urban area should have a minimum population concentration of 1,000 and a population density of at least 400 persons per square kilometre based on the previous census population counts (Statistics Canada, 1996). Many countries impose much higher thresholds, such as Greece, with 10,000 inhabitants and Japan with 30,000 (Herbert and Thomas, 1997). The diversity of this threshold largely relates to the social context of the country. For instance, considering the physical geography of Scandinavia and the ways in which its settlements evolved over time, a settlement with over 200 permanent inhabitants may well be regarded as urban. On the other hand, in a country like Japan with a relatively limited land area and considerable population pressure, almost all settlements exceed such a low threshold of 200 inhabitants and a threshold of 30,000 inhabitants seems more realistic in delimiting its urban extent (Herbert and Thomas, 1997).

In addition to the use of the combination of population size and density, a pure criterion of population density was also applied in delimiting urban areas. Gryztzell (1963) presented a method based on population densities alone. He argued that fair comparison could only be made when the densities involved were similar. He therefore attempted to delineate areas where minimum densities could be equated using the smallest administrative unit. By calculating the population densities, he worked outward from the large cities until points were reached where the densities fell below a given figure. This allows a line to be drawn around the city so that all areas with a density over a given threshold were included. Gryztzell (1963) identified a series of threshold values of this density and used them for comparative purposes. However, his approach overlaps with his definition of delimiting urban area using population density as a sole criterion (Carter, 1976).

Australia is a country with high urban population and extensive living space. By the early 1970s, 86 percent of all Australians lived in towns or cities (Frost and Dingle, 1995). However, all capital cities developed their suburbs before the centres were fully built-up, in the hope of ‘creating healthy and spacious living conditions’ (Frost and Dingle, 1995:20). By the start of the twentieth century Australia’s major cities had either developed a low-density townscape with significant decentralization of housing and jobs, or were beginning to sprawl at the edge of their old compact cores. Compared to the concentrated cities of Europe and eastern North America, these cities were of remarkably low density and cover immense areas of ground (Frost and Dingle, 1995). Under this background, the criteria for delimiting its urban areas in relation to population size and density are quite low. The delimitation criteria for urban centres currently in force in Australian Standard Geographical Classification (ASGC) are based on those developed by Linge (1965) with subsequent amendment by the Conferences of Statisticians of Australia in 1965 and 1969 and the Review of the Australian Bureau of Statistics (ABS) Statistical Geography in 1988 (ABS, 1999). The core points of these criteria are as follows: urban centres with a population of 20,000 or more consist of a cluster of contiguous urban census collector’s districts (CCDs) and other urban areas. CCDs classified as urban include the following:

- All contiguous CCDs which have a population density of 200 or more persons per square kilometre shall be classified as urban;
- A CCD consisting mainly of land used for factories, airports, small sports areas, cemeteries, hostels, institutions, prisons, military camps or certain research stations shall be classified as urban if contiguous with CCDs which are themselves urban;
- A CCD consisting mainly of land used for large sports areas, large parks, explosives handling and munitions areas, or holding yards associated with meatworks and abattoirs shall be classified as urban only if it is bordered on three sides by CCDs which are themselves classified as urban;
- Any area which is completely surrounded by CCDs which are urban must itself be classified as urban.

For urban centres with a population between 1,000 and 19,999, their urban boundaries are delimited subjectively by the inspection of aerial photographs, by field inspection and/or by consideration of any other information that is available. All contiguous urban growths are to be included (even though these would not necessarily occur if the density criterion were applied), together with any close but non-continuous development which could be clearly regarded as part of the urban centre. However, for urban centres which contain a population approaching 20,000 the objective criteria applied for urban centres with 20,000 people should also be considered (ABS, 1999).

More recently, with the development of remote sensing technology, various researchers have examined applications of remotely sensed data for identification of urban land-cover characteristics and their change over time (Ward, Murray & Phinn, 2000; Hepner, *et al.*, 1998; Barnsley and Barr, 1997, 1996; Harris and Ventura, 1995; Mesev, *et al*., 1995; Forster, 1993, 1983; Gong and Howarth, 1990; Moller-Jensen, 1990). Five recurrent research themes have been identified in this regard, which are: 1) the delimiting of land-cover and land use types; 2) assessment of the utility of texture measures to aid in separating urban land-cover and land use types; 3) mapping areas of impervious and pervious surfaces for input into energy and moisture flux models; 4) mapping land-cover and land use change in urban area; and 5) application of empirical models to estimate biophysical, demographic and socio-economic variables (Phinn, *et al*., 2001). Although these researchers have met mixed success, their approaches were developed based on classical set theory. In other words, they understood the boundary between an urban area and its non-urban hinterland as a sharp line.

As cities are physical concentrations of population and housing, population or dwelling densities are measurable criteria in delimiting the extent of urban development. Therefore, it is appropriate to use population and/or dwelling densities as criteria to delimit urban areas. In particular, these criteria allow for the application of fuzzy set theory to define a fuzzy concept of urban areas. In this thesis, instead of defining a sharp boundary between urban and non-urban areas, the fuzzy set approach to delimiting the extent of urban development with population density is proposed. These criteria are flexible and they can be adjusted or calibrated according to different conditions when applied to individual cities.

Fuzzy membership function in delimiting urban areas

To define a fuzzy membership function for delimiting urban areas in this paper, a population density criterion was employed. This population density value has been adjusted based on a number of other factors, such as dwelling density, major infrastructure such as sewerage and drainage supply, type of land use depicted from satellite images and the percentage of population dependent on non-agricultural industries. For example, if an area has a low population density but the dwelling density is high or if an area has intensive type of urban land use, the population density needs to be adjusted to a higher value. In contrast, if an area has a relatively high population density but most of its land is used as farms, the population density value needs to be reduced to some extent. Such factors were evaluated when applying the model to simulate urban development of metropolitan Sydney.

A common approach for defining urban areas is evident from various countries using census data, i.e., an area is regarded as urban if it reaches a certain value of population density. Due to the widespread adoption of this approach the assumption was also applied in this paper: if an area has a population density less than a certain value, it is regarded as non-urban, and the grade of membership of this area in the urban fuzzy set is 0. If an area has a population density higher than another threshold value, it is regarded as fully urban built-up area, and therefore its membership grade is 1. The lower and upper threshold values of population density can vary significantly from one country to another, or even from one city to another. Therefore, these threshold values need to be defined individually according to situations in different countries or cities. For example, the lower threshold of population density excluding an area from being regarded as urban can be 200 person/km^2 in Australia, while this threshold in Canada and the U.S. should be 400 person/km^2 and 580 person/km^2 respectively. With these threshold values, the membership function of areas in the urban fuzzy set can be defined as a function of its population density.

Let r_0 and r_1 be the lower and upper thresholds of population density respectively in delimiting urban areas. A simple linear membership function was employed, as is shown in Equation 3.4:

$$m_{urban}(x_{ij}) = \begin{cases} 0 & r_{x_{ij}} < r_0 \\ \dfrac{r_{x_{ij}} - r_0}{r_1 - r_0} & r_0 \le r_{x_{ij}} < r_1 \quad (x_{ij} \in X) \\ 1 & r_{x_{ij}} \ge r_1 \end{cases} \tag{3.4}$$

With this membership function, the population density values can be converted into the grade of membership. This involves matching the density measurement against the membership function (Figure 9), which is called a fuzzification process (Berenji, 1992). Through this conversion, each cell in the urban fuzzy set receives one value of membership grade, which represents the state of the cell in the urban fuzzy set. For instance, if the population density of a cell is less than the lower threshold, it receives a membership grade of 0. In this case, the state of the cell is regarded as ‘non-urban’. If the population density of a cell is higher than the upper threshold, it receives a membership grade of 1, the state of which is regarded as ‘urban’. All other cells receive a membership grade between 0 and 1, representing their extent of development in this urban fuzzy set. Their states in the urban fuzzy set are termed ‘partly-urban’. Therefore, instead of using a binary definition of non-urban and urban, multiple states in delimiting urban areas can be applied to simulate the continuous process of non-urban to urban conversion.

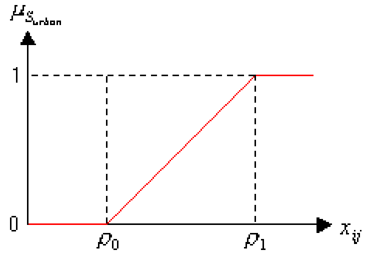


Figure 9: Matching a density measurement with the membership function

$m_{S_{urban}}$ is the membership function of cell x_{ij} in the urban fuzzy set; r_0 and r_1 are the lower and upper thresholds of population density respectively in delimiting urban areas.

3.3 Fuzzy logic control in cellular automaton urban modelling

Urban development as a fuzzy process is not only represented in delimiting its urban extent, it is also represented in the factors driving such development. As a result of both physical constraints and human decision-making behaviours, urban development is a fuzzy logic controlled process. This section introduces the methodology of fuzzy logic control in the simulation of urban development. As fuzzy logic control is developed based on two important concepts - the linguistic variables and fuzzy logic, these concepts are discussed first, followed by discussions on the application of fuzzy logic control in urban modelling, especially in models using cellular automaton approach.

Linguistic variables

'In retreating from the precision in the face of overpowering complexity, it is natural to explore the use of what might be called *linguistic* variables, that is, variables whose values are not numbers but words or statements in a natural or artificial language' (Zadeh, 1973:9). An easy way to understand the notion of a linguistic variable is to regard it as a variable whose numerical values are fuzzy numbers or as a variable the range of which is not defined by numerical values but by linguistic terms. Zadeh (1973:75) provided a formal definition of linguistic variables as follows:

> A linguistic variable is characterized by a quintupe (x, T(x), U, G, $\tilde{M}$) in which x is the name of the variable; T(x) (or simply T) denotes the term set of x, that is, the set of names of linguistic values of x, with each value being a fuzzy variable denoted generically by x and ranging over a universe of discourse U which is associated with the base variable u; G is a syntactic rule (which usually has a grammatical form) for generating the name, X, of values of x; and $\tilde{M}$ is a semantic rule for associating with each X its meaning, $\tilde{M}$ (x), which is a fuzzy subset of U. A particular X - that is, a name generated by G - is called a term (Zadeh, 1973:75).

There are some special linguistic terms such as *very*, *more or less*, *fairly* and *extremely* which modify the meaning of other linguistic terms. These are called linguistic hedges (or simply hedges or modifiers). Mathematical models frequently used for modifiers include

Concentration: $m_{\text{con}(\tilde{A})}(u) = (m_{\tilde{A}}(u))^2$ (3.5)

Dilution: $m_{\text{dil}(\tilde{A})}(u) = (m_{\tilde{A}}(u))^{1/2}$ (3.6)

Contrast intensification: $m_{\text{int}(\tilde{A})}(u) =$

$$\begin{cases} 2(m_{\tilde{A}}(u))^2 & \text{for } m_{\tilde{A}}(u) \in [0,0.5] \\ 1-2(1-m_{\tilde{A}}(u))^2 & \text{otherwise} \end{cases} \quad (3.7)$$

Fuzzy logic and fuzzy logic control

Fuzzy logic (Zadeh, 1973) is an extension of classical formal models of reasoning into models that incorporate fuzziness. The fundamental difference between classical logic and fuzzy logic is in the range of their truth-values. In fuzzy logic, the truth or falsity of fuzzy proposition is a matter of degree. Assuming that truth and falsity are expressed by values 1 and 0 respectively, the degree of truth of each fuzzy proposition is expressed by a number in the unit interval of [0, 1]. In fuzzy logic, the number of truth-values is, in general, infinite. Instead of using numbers, the truth-values are linguistic variables (or terms of the linguistic variable truth). The terms of the linguistic variable 'truth' can be tabulated as a finite number of terms, such as true, very true, false, more or less false, very false, and so on. With this tabulation, the logic operators, like 'and', 'or' and 'not' are also defined in fuzzy logic, and the extensive principles can be applied to derive definitions of these operators.

Fuzzy logic provides 'a means of translating natural language-based expressions of knowledge and common sense into a precise mathematical formalism' (Openshaw and Openshaw, 1997:269). It also gives computers the ability to think and make decisions more like human beings (McNeill and Freiberger, 1994). For geographers, it offers 'a refreshingly new perspective on how to go about building better models of geographical systems by handling rather than ignoring or artificially removing the fuzziness within them' (Openshaw and Openshaw, 1997:269).

Fuzzy logic control is the application of fuzzy set theory and fuzzy logic in control systems. The basic idea behind this approach is to incorporate the 'experience' of a human process operator in the design of the controller. From a set of linguistic rules which describe the operator's control strategy a control algorithm is constructed where the words are defined as fuzzy sets. The main advantages of this approach seem to be the possibility of implementing 'rule of thumb' experience, intuition and heuristics and the fact that it does not need a model of the process (Kickert and Mamdani, 1978). In a fuzzy logic control system, fuzzy logic is used to convert heuristic control rules as stated by a human operator into an automatic control strategy (Mamdani and Assilian, 1975).

For example, the development of an area can be controlled by a set of linguistic rules like the follows:

IF	the slope of an area is very high
AND	the infrastructure system of the area is not complete
THEN	development in this area should be very slow

In the development of a fuzzy logic control system, different methods have been suggested over the last twenty years. A typical fuzzy logic control system consists of four modules: a fuzzy rule base, a fuzzy inference engine, a fuzzification module and a defuzzification module (Figure 10).

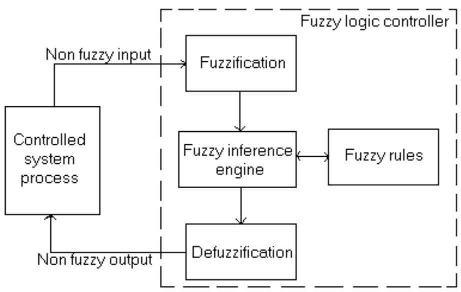


Figure 10: The fuzzy logic control system

In a fuzzy logic control system, both the input and the output have measured non-fuzzy values. To design a fuzzy controller, one must identify the main control parameters and determine a term set using linguistic variables to convert the measurements of input into appropriate fuzzy sets to express measurement uncertainties. This process is called fuzzification. The fuzzified measurements are used by the fuzzy inference engine to evaluate the control rules in the fuzzy rule base. The result of this evaluation is a fuzzy set representing possible control actions. The fuzzy set needs to be converted into a crisp set, which should be considered as the representation of the fuzzy set. This conversion process is called defuzzification (Berenji, 1992). The defuzzified value (or a vector of values) represents actions taken by the fuzzy controller of the system.

Fuzzy set and fuzzy logic control provide a linguistic non-numerical, non-mathematical and non-statistical based approach to modelling complex systems, and it is a fairly simple approach with few rules being required to handle considerable complexity. This approach has found much practical applicability in industries and engineering systems (for example, Umbers and King, 1980; Kickert and Van Nanta Lemka, 1976; Mamdani and Assilian, 1975). It has also been applied in social sciences, especially in simulating the human decision-making processes (e.g., Kickert, 1978; Wenstøp, 1976).

Fuzzy logic control in cellular automaton urban modelling

A cellular automaton is a dynamic system in which the state of cells of the system is determined by the state of the cell itself and states of cells of its neighbourhood at the previous time step based on certain transition rules. In this system, the rules controlling the transition of cells from one state to another can be deterministic, such as rules implemented in the 'Game of Life' (Gardner, 1972). However, the transition rules in a cellular automaton can also be non-deterministic, which are more appropriate when a cellular automaton represents a human-related system. For example, in an urban system, although it is understood that topography is a constraint to urban development, it is not correct to claim that '*an area with a slope higher than 20 degrees cannot be developed into an urban area*'. Even though areas with smooth terrain can be first selected for development, development could occur in areas with steep terrain under certain circumstances, such as high demand for land but short supply of flat land, or the high income with people preferring to live on higher elevations to obtain good views. Therefore, the terrain constraint is not a deterministic factor to urban development functioning on a 'yes' or 'no' base.

Urban development is a complex spatial phenomenon controlled by many factors. The geographical conditions of the area, socio-economic status, infrastructure supply, demographic features and the potential of population growth, planning and zoning constraints, environmental protection regulation as well as group and individual behaviour all play a role in the process of urban development. However, none of these factors functions in a deterministic manner. In other words, urban development is not controlled by Boolean logic. Instead, the controlling process of these factors is based on fuzzy logic. For instance, although it is not appropriate to say that '*an area with a slope of 20 degrees cannot be developed into an urban area*', it will be true to say that 'development is *less likely* to happen to steep terrain land'. With the use of a linguistic variable, the rule becomes fuzzy, and it functions according to the regulation of fuzzy logic. Therefore, it is believed that advantages would come by incorporating fuzzy logic control in a cellular automaton to simulate the process of urban development.

4. DEVELOPMENT OF A FUZZY-LOGIC - CONTROLLED CELLULAR AUTOMATON MODEL OF URBAN DEVELOPMENT

Before introducing the fuzzy logic controlled transition rules in a cellular automaton model to simulate the process of urban development, the following two assumptions were proposed:

1). The state 'urban' is the highest state a cell can achieve in the urban fuzzy set. Therefore, no re-development or urban consolidation process is considered in this model;

2). Development can only happen from a lower state to a higher one, i.e., from non-urban to partly-urban to urban. No anti-urbanization process is taken into consideration in this model.

4.1 Logistic curve of urban development

Previous research demonstrates that the process of urban development follows a logistic curve over time (Herbert and Thomas, 1997; Jakobson and Prakash, 1971; Fourastié, 1963). Fourastié (1963) suggests that the period of industrialization and urbanization is a transitory stage in the history of mankind during which societies transform from primary or agriculture-based stage to tertiary or service-occupation based stage (Figure 11). The transition stage is divided into three parts labelled take-off, expansion and achievement. The process of tertiary civilization is a logistic curve, progressing from 10 to 80 percent levels of urbanization in a society. Although the three-sector theory has its shortcomings (Courtheaux, 1969), it demonstrates a general trend of the process of urban development. This logistic trend of development has been identified in many places around the world (Figure 12).

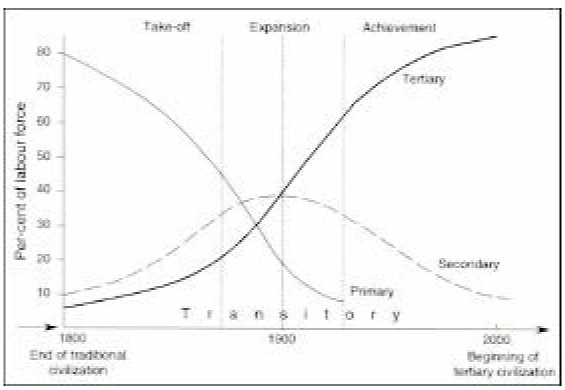


Figure 11: Urban development and the progression from transitional to modern
(Source: Herbert and Thomas, 1997:41, reproduced with the authors' and the publisher's permission)

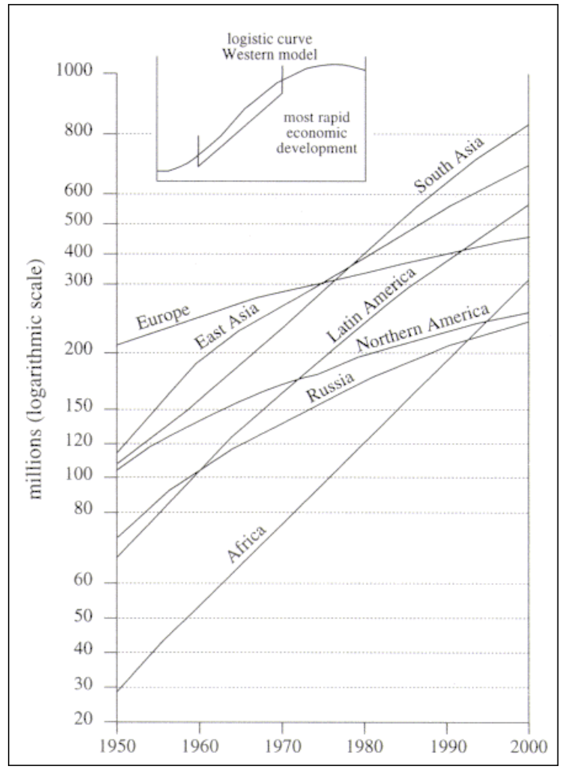


Figure 12: Logistic curve of urban development between 1950 and 2000
Y axis is population in million; X axis is time.
(Source: Herbert and Thomas, 1997:45, reproduced with the authors' and the publisher's permission)

Assuming that the full process of urban development in an area (or a cell) takes n years, i.e., if a non-urban cell starts a process of urban development, this cell will be fully urbanized after n years of development. Let $t_{x_{ij}}$ denote the t[th] year of development of a cell x_{ij}. The extent of urban development of the cell in year t can be represented by a membership grade, which can be denoted by $m_{S_{urban}}(x_{ij}^t)$. With the understanding of the logistic curve of the process of urban development, the relationship between $t_{x_{ij}}$ and $m_{S_{urban}}(x_{ij}^t)$ can be represented by Equation 4.1.

$$m_{S_{urban}}(x_{ij}^t) = \begin{cases} 0 & t_{x_{ij}} < 0 \\ \dfrac{1}{a_0 + b_0 \exp(-c_0 \cdot t_{x_{ij}})} & 0 \le t_{x_{ij}} < n \\ 1 & t_{x_{ij}} \ge n \end{cases} \tag{4.1}$$

where a_0, b_0 and c_0 are parameters of the logistic curve; n is the duration of the whole process of urban development. The shape of the growth curve usually represents the speed of urban development over time, which is controlled by three parameters, i.e., a_0, b_0 and c_0. The speed of urban development varies from one city to another, and even from one cell to another with a city. However, the shape of the growth curve is not very sensitive to changes made through parameters a_0 and b_0; it is sensitive to changes made through parameter c_0. Therefore, a_0, b_0, c_0 and n can be defined and calibrated according to the speed of urban development of individual cities.

4.2 Incorporating fuzzy-logic-controlled transition rules

Generic mode of urban development

According to the principles of cellular automata, the state of the cell itself and the states of its neighbouring cells at a previous time step determine the state of a cell in the urban fuzzy set. If a cell has a strong propensity for development and it can get support for such development from its neighbourhood, then development will occur to that cell. For instance, if a cell has a membership grade larger than a certain value at time t in the urban fuzzy set, which means it has a strong propensity for development, and the average grade of membership of its neighbouring cells is larger than the membership grade of the cell itself, which means there exists a driving force for development from its neighbourhood, the cell will undergo further development following the logistic curve illustrated in Equation 4.1.

To determine what state a cell will be in after a certain time period, Equation 4.1 can be rewritten as Equation 4.2:

$$t_{x_{ij}} = \begin{cases} 0 & t_{x_{ij}} < 0 \\ (\ln(b_0) - \ln(m^{-1}_{S_{urban}(x^t_{ij})} - a_0))/c_0 & 0 \le m_{S_{urban}(x^t_{ij})} < n \\ n & t_{x_{ij}} \ge n \end{cases} \tag{4.2}$$

For each cell x_{ij} in the urban fuzzy set, its membership grade has been defined through its population density value in Equation 3.4. Therefore, the stage (or year as denoted by $t_{x_{ij}}$) of development of the cell in the urban development process can be calculated in Equation 4.2.

With the awareness of the current stage of development of cells in the urban development process, the grade of membership of this cell in the urban fuzzy set at another time t' can then be computed through Equation 4.1.

Constrained patterns of development

However, not all cells in the urban system are developing at the same time or at the same speed. Variation of the state of cells and factors such as the geographic conditions of the cell and its neighbouring cells, socio-economic status, planning and government polices can have significant impacts on such development, resulting in variation in the pattern of urban development in space and over time. For instance, if a cell has the propensity for development but it cannot get sufficient support from its neighbourhood, development could be slowed down. This slow development can also occur if the cell sits in a high terrain or a deep slope area. With the support of transportation networks, development might be expedited. Cells which are sited in water bodies, such as sea, lakes or rivers, or cells which are located in areas reserved for various purposes cannot be developed. In some areas at some stage, advantageous conditions for development could lead to new development in undeveloped non-urban areas. For instance, with the construction of a new railway line extending from an urban to a rural region, areas along the new railway line could be selected for urban development. Based on fuzzy logic control, the basic pattern of development can be modified using a number of linguistic variables such as ‘quick’, ‘slow’, ‘very quick’ or ‘very slow’ and so on to achieve different scenarios of development. Based on the definition of the generic mode of urban development, the following four membership functions were proposed to represent various constrained patterns of development.

Basic pattern of development

$$m_{S_{urban}}(x^{t'}_{ij}) = \begin{cases} 0 & t'_{x_{ij}} < 0 \\ \dfrac{1}{a_0 + b_0 \exp(-c_0 \cdot t'_{x_{ij}})} & 0 \le t'_{x_{ij}} < n \\ 1 & t'_{x_{ij}} \ge n \end{cases} \qquad (c_0 = 0.5) \tag{4.3}$$

Quick development

$$m_{S_{urban}}(x^{t'}_{ij}) = \begin{cases} 0 & t'_{x_{ij}} < 0 \\ \dfrac{1}{a_0 + b_0 \exp(-c' \cdot t'_{x_{ij}})} & 0 \le t'_{x_{ij}} < n \\ 1 & t'_{x_{ij}} \ge n \end{cases} \qquad (c' > 0.5) \tag{4.4}$$

Slow development

$$m_{S_{urban}}(x^{t'}_{ij}) = \begin{cases} 0 & t'_{x_{ij}} < 0 \\ \dfrac{1}{a_0 + b_0 \exp(-c' \cdot t'_{x_{ij}})} & 0 \le t'_{x_{ij}} < n \\ 1 & t'_{x_{ij}} \ge n \end{cases} \qquad (0 < c' < 0.5) \tag{4.5}$$

New development

$$m_{S_{urban}}(x^{t'}_{ij}) = \sqrt{m_{S_{urban}(x^t_{ij})}} \tag{4.6}$$

No development

$$m_{S_{urban}}(x^{t'}_{ij}) = m_{S_{urban}}(x^t_{ij}) \tag{4.7}$$

Through the variation of parameters c' in Equations 4.4 and 4.5, more scenarios of urban development such as ‘Very quick development’, ‘Extremely quick development’, ‘Very slow development’ and ‘Extremely slow development’ can be implemented in the model. The linguistic variables used in this paper are defined as follows:

Quick: $c' = (1 + \frac{1}{4}) \times c_0$

Very quick: $c' = (1 + \frac{1}{2}) \times c_0$

Extremely quick: $c' = (1 + \frac{3}{4}) \times c_0$

Slow: $c' = \frac{3}{4} \times c_0$

Very slow: $c' = \frac{1}{2} \times c_0$

Extremely slow: $c' = \frac{1}{4} \times c_0$

Figure 13 illustrates the membership trends of seven different patterns of development in an urban system, based on a 20-year period for full development.

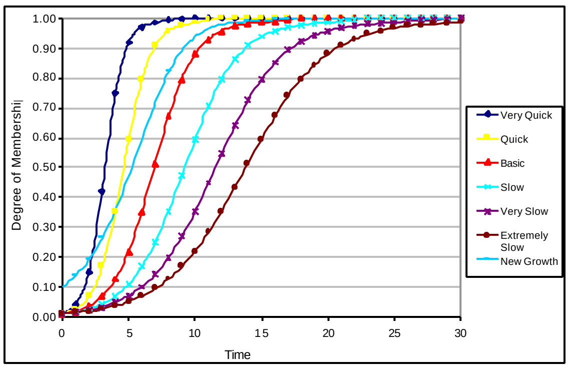


Figure 13: Seven different patterns of urban development

Fuzzy inference engine

In modelling of urban development based on cellular automaton principles, each of the patterns of development described in Equations 4.1 through 4.6 is regarded as a transition. For the development of a cell in this system, the question of which transition rule applies at a certain time depends on the condition of the cell itself and the conditions of cells in its neighbourhood. These conditions can be either physical, socio-economic, or institutional, or the combination of any or all.

Let T_1, T_2, T_3, T_4, T_5, T_6, T_7 and T_8 represent eight different transitions in the development of a cellular automaton based urban system. These transitions are termed as 'Normal development' (basic pattern) (T_1), 'Slow development' (T_2), 'Very slow development' (T_3), 'Extremely slow development' (T_4), 'Quick development' (T_5), 'Very quick development' (T_6), 'New development' (T_7) and 'No development' (T_8) respectively. The basic pattern of development (T_1) will be implemented if a cell has a strong propensity for development and it can also get sufficient support for such development from its neighbourhood. No other constraints such as topographic variations or transportation networks or planning schemes apply to the cell. If the state of cells at the neighbourhood of a cell x_{ij} is represented by the average grade of membership of all the neighbouring cells in the urban fuzzy set, which is denoted by $\bar{m}_{S_{urban}}(\Omega_{x_{ij}})$, then the rule under which T_1 applies can be expressed as:

Condition A $m_{S_{urban}}(x_{ij}) >= m_0$ and

$$\bar{m}_{S_{urban}}(\Omega_{x_{ij}}) > m_{S_{urban}}(x_{ij}) \quad (4.8)$$

where m_0 is a minimum threshold delimiting a cell with strong propensity for development. For example, m_0 can be set to 0.5 for the first instance and it can be calibrated within the model on the simulation process.

If a cell has a strong propensity for development but it cannot get sufficient support for such development from its neighbourhood, a slow development pattern (T_2) applies to that cell. This slow development pattern also applies if a cell does not have a strong propensity for development but it can get support for development from its neighbourhood. Therefore, T_2 applies if

Condition B $m_{S_{urban}}(x_{ij}) >= m_0$ and

$$\bar{m}_{S_{urban}}(\Omega_{x_{ij}}) < m_{S_{urban}}(x_{ij}) \quad (4.9)$$

or Condition C $0 < m_{S_{urban}}(x_{ij}) < m_0$ and

$$\bar{m}_{S_{urban}}(\Omega_{x_{ij}}) > m_{S_{urban}}(x_{ij}) \quad (4.10)$$

Similarly, if a cell has a weak propensity for development and it cannot get sufficient support for such development from its neighbourhood, a very slow development process applies to that cell. Therefore, T_3 applies if

Condition D $0 < m_{S_{urban}}(x_{ij}) <= m_0$ and

$$\bar{m}_{S_{urban}}(\Omega_{x_{ij}}) < m_{S_{urban}}(x_{ij}) \quad (4.11)$$

Apart from the above situation, if a cell does not have the propensity for development and it cannot obtain support for such development from its neighbourhood, then no development can occur in that cell. Therefore, T_8 (No development) applies. Moreover, a cell may sit in areas where urban development is strictly constrained either by physical conditions or by institutional concern. In this case, no development will occur in that cell regardless of the state of the cell itself and the states of its neighbouring cells.

As urban development can be sped up or slowed down by a number of factors such as topographic conditions, transportation networks, socio-economic status as well as planning and human decision-making behaviour, the above four transitions (T_1, T_2, T_3 and T_8) can be applied under the impact of these factors. In addition, more transition rules such as T_4 to T_7 can be applied. For instance, if the topographic constraint is introduced into the model, then T_2 applies if

- A cell is located in a *high* slope area; and
- It has a *strong* propensity for development; and

- It can also get *sufficient* support for development from its neighbourhood.

Similarly, T_3 applies if

- A cell is located in a *high* slope area; and
- It has a *strong* propensity for development, but it cannot get *sufficient* support for development from its neighbourhood; or
- It has a *weak* propensity for development, but it can get *sufficient* support for development from its neighbourhood.

T_4 applies if

- A cell is located in a *high* slope area; and
- It has a *weak* propensity for development; and
- It cannot get *sufficient* support for development from its neighbourhood.

And T_7 applies if

- An undeveloped cell is sited in a very flat area; and
- It can get *sufficient* support for development from its neighbourhood.

Considering the impact of transportation networks on urban development, more rules can be implemented in addition to the rules demonstrated above in the transition of such a cellular automaton based urban system. For instance, T_1 applies if

- A cell is in an area with convenient transportation systems; and,
- It can get *sufficient* support for development from its neighbourhood; however
- The propensity for development from the cell itself is *weak*.

T_2 applies if

- A cell is in an area with convenient transportation systems; however
- It has a *weak* propensity for development; and
- It cannot get *sufficient* support for development from its neighbourhood.

T_5 applies if

- A cell is in an area with convenient transportation systems; and
- It has a *strong* propensity for development; and
- It can also get *sufficient* support for development from its neighbourhood.

More constraints can be introduced into the model individually or in combination with one another through the above procedure. By calibrating the model against the actual process of urban development, it is convenient for modellers to evaluate the impact of each factor on the process of urban development.

4.3 Implementing the model in a GIS

Cellular automata models can be implemented within many types of software (Batty, 1997). For cellular automata models of urban development, programs have been implemented both inside and outside a GIS environment. For the former type, Itami and Clark (1992) and Itami (1988) developed their models within a raster GIS of Idrisi and MAP II, and Wu (1998a, 1998b, 1998c) implemented his model using ARC/INFO's Arc Macro Language (AML). These programs took the advantage of the graphic capabilities and the friendly user interface of a GIS. Other programs were developed outside a GIS, although most were loosely coupled with a GIS for data manipulation and visualisation. This kind of models can be exemplified by Batty, Xie and Sun (1999), Batty (1998), Clarke, Hoppen and Gaydos (1997). As software packages become more open to one another, and they are incorporating more generic programming capabilities and graphic functions, 'dedicated software for developing CA becomes less important' (Batty, 1997:272).

Nevertheless, geographical models developed within a GIS are still preferable in many applications. In this project, the model is programmed using ARC/INFO's AML in a GRID environment. There are a number of reasons for using such a strong coupling strategy. One is that data are processed and stored in the GIS as grid files, and the simulated output of the model are also stored as grids in the GIS. No data conversion is necessary between the GIS and the model, which saves time on data communication and conversion. This feature is especially advantageous during the model calibration process, when simulation results need to be compared and fitted with data illustrating actual urban development. Another useful feature of the model is its spatial visualisation capability. As all input data and output results are stored as ARC/INFO grids, these data can easily be visualised spatially in either the ARC/INFO or ArcView programs. Calibration of the model and analysis of the goodness-of-fit of the model's output with the actual urban development can be conducted easily using the GRID tools in ARC/INFO or the spatial analysis module in ArcView. In addition, by creating a friendly user interface of the model using the AML, parameters used in the model can be provided and controlled by the user, so as the implementation of different transition rules and different scenarios of development.

4.4 Experimental scenarios of development

Before the application of the cellular automaton model to a real city, it was applied to simulate the process of urban development of a simplified artificial urban system for the easy identification of the model's behaviour. For this purpose, a cellular automaton based urban system was proposed, and consisted of 100 by 100 cells. The states of cells in the urban development process were defined artificially (Figure 14(a)). With the assumption that a full process of urban development takes 20 years, i.e., n = 20 in Equation 4.1, the model first simulates a scenario of development under neither physical nor institutional constraints. Development is controlled only by the state of a cell itself and the states of its neighbouring cells. In this situation, only cells with a membership grade larger than 0 have the opportunity to develop. In other words, development occurs to cells that have already started their development process. The higher the grade of the membership, the quicker the cell is developed. No development occurs to the undeveloped cells whose membership grade is 0, as these cells were supposed to have neither self-propensity for development nor can they obtain support for development from their neighbourhood. Hence, the urban area would not expand to the non-urban area over time. Only four transition rules were applied to control the process of this development. These rules are normal development (T_1), slow development (T_2), very slow development (T_3) and no development (T_8). Figure 14 (b), (c) and (d) demonstrate the results of development of this city at the fifth, tenth and the twenty-fifth years respectively.

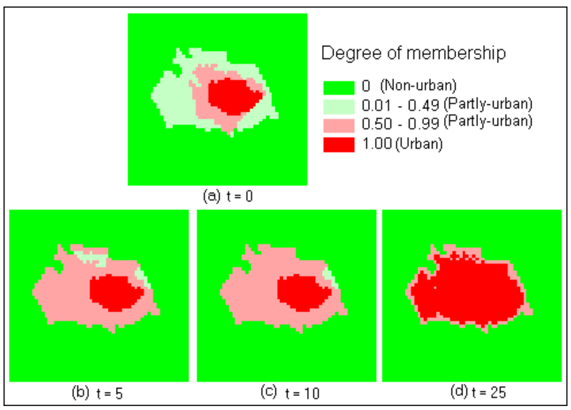


Figure 14: Unconstrained urban development a) initial states of the city; b) urban scenario after five years; c) urban scenario after ten years; d) urban scenario after 25 years. Circular neighbourhood applies, radius = 1.

If the topographic condition of the city varies from place to place, then this factor needs to be introduced into the model as a constraint. As development can be expedited in areas with smooth terrain and flat slope, and it can be slowed down in areas with high terrain and steep slope, more rules can be implemented on the process of the urban development. These include transitions T_4 and T_7 in addition to the four transitions applied above. Figure 15 (a) and (b) demonstrate the topographic condition of the city, which were defined artificially, and Figure 15 (c) to (f) demonstrate scenarios of development under these conditions at different iterations.

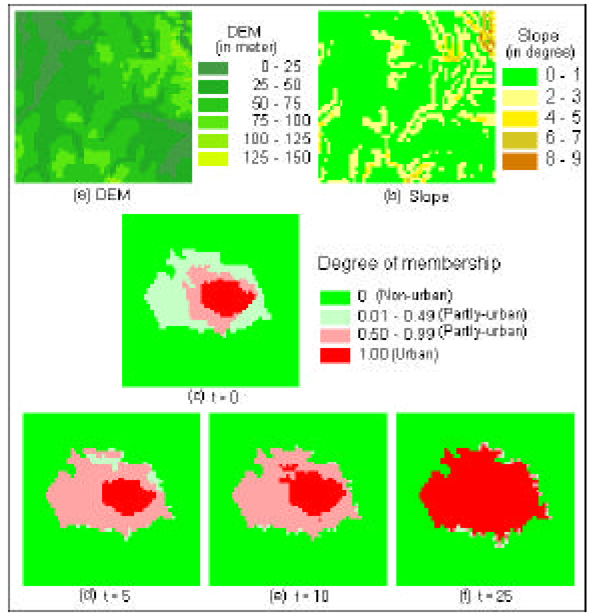


Figure 15: Topographic constrained urban development
a) DEM of the city; b) slope; c) initial states of the city; d) urban scenario after five years; e) urban scenario after ten years; f) urban scenario after 25 years. Circular neighbourhood applies, radius = 1.

In addition to the topographic constraint, other factors such as the transportation networks can also have important impacts on the process of urban development of this city. Figure 16 demonstrates the results of development of the city under both the topographic constraint and the transportation networks. In this situation, development has been attracted significantly to areas along the major transport routes.

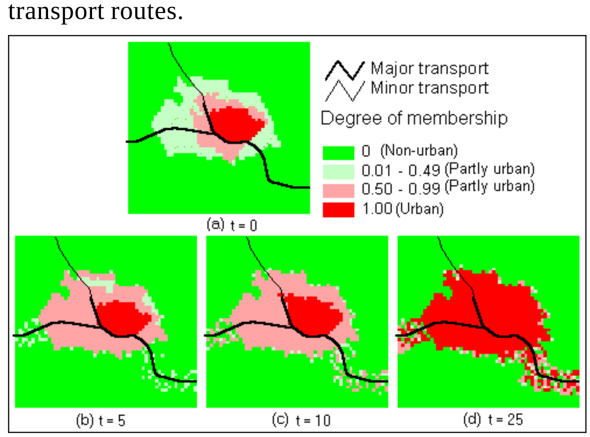


Figure 16: Topographic constrained and transportation supported urban development (See Figure 14 for notes)

Experimental application of the model to an artificial city demonstrates that the model constructed based on the cellular automata principles and incorporating fuzzy set approaches is capable of generating realistic scenarios of urban development. Urban development was controlled by various transition rules which reflect actual conditions of the city being modelled. The simple application of rules demonstrates the validity of the model. It also indicates that more complicated rules including urban infrastructure supply, socio-economic status as well as planning and government policies can be implemented into the model to evaluate how and to what extent these factors affect the development of the city.

5. CONCLUSIONS

This paper developed a model of urban development based on the principles of cellular automata and incorporating fuzzy set approaches. Compared with other urban models based on cellular automata principles, this model possesses advantageous features for simulating urban development process. One of these features is the incorporation of fuzzy set theory and fuzzy logic in the definition of cell states and the transition rules. With fuzzy set theory, the state of a cell is associated with a grade of membership representing the stage a cell is in its urban development process. The grade of membership represents urban development as a continuous process in space and over time, rather than as a binary non-urban to urban conversion process. In addition, the use of fuzzy transitions such as 'quick development', 'very quick development', 'slow development', 'very slow development', 'extremely slow development' and various linguistic variables such as 'good', 'sufficient', 'not so sufficient' makes the definition of transition rules more close to human decision-making behaviour.

Advantages for modelling urban development also come from integrating the model in a geographical information system. With this integration it is easy to control the performance of the model, visualise its output and calibrate it. Experimental application of the model to an artificial city produced realistic patterns of development supporting the modelling approach. A more appropriate and rigorous test of the model is required to validate its approach and output. The follow-on from this paper will describe the results obtained from applying the model to Sydney, Australia.

REFERENCES

ABS - Australian Bureau of Statistics Statistical Concepts Library (1999) "Australian Standard Geographical Classification (ASGC) 1999. Chapter 6. Urban Centre/Locality (UC/L) Structure - The spatial units", Hp. Online. Available: http://www.abs.gov.au/ausstats/abs%40.nsf/66f306f503e529a5ca25697e0017661f/29d2969904a69ffcca25697e00184c33!OpenDocument. 15 July 2001.

Barnsley, M. and Barr, S. (1997) "Distinguishing urban land-use categories in fine spatial resolution land-cover data using a graph-based, structural pattern recognition system", *Computers, Environment & Urban Systems*, vol. 21(3/4), pp.209-225.

Barnsley, M. and Barr, S. (1996) "Inferring urban land-use from satellite sensor images using kernel-based spatial re-classification", *Photogrammetric Engineering and Remote Sensing*, vol. 62, no. 8, pp.949-958.

Batty, M. (2000) "GeoComputation using cellular automata", in *GeoComputation*, eds. S. Openshaw and R. J. Abrahart, Taylor & Francis, London, pp.95-126.

Batty, M. (1998) "Urban evolution on the desktop: simulation with the use of extended cellular automata", *Environment and Planning A*, vol. 30, pp.1943-1967.

Batty, M. (1997) "Cellular automata and urban form: a primer", *Journal of the American Planning Association*, vol. 63, pp.266-274.

Batty, M. and Xie, Y. (1997) "Possible urban automata", *Environment and Planning B*, vol. 24, pp.175-192.

Batty, M. and Xie, Y. (1994) "From cells to cities", *Environment and Planning B*, vol. 21, pp.31-48.

Batty, M., Couclelis, H. & Eichen, M. (1997) "Urban systems as cellular automata", *Environment and Planning B*, vol. 24, pp.59-164.

Batty, M., Xie, Y. & Sun, Z. (1999) "Modelling urban dynamics through GIS-based cellular automata", *Computers, Environment and Urban Systems*, vol. 23, pp.205-233.

Berenji, H. R. (1992) "Fuzzy logic controller", in *An Introduction to Fuzzy Logic Applications in Intelligent Systems*, eds R. R. Yager and L. A. Zadeh, Kluwer Academic Publishers, London.

Bryant, C. R., Russwurm, L. H. & McLellan, A. G. (1982) *The city's countryside: land and its management in the rural-urban fringe*, Longman, New York.

Burrough, P. A. (1986) *Principles of geographic information systems for land resources assessment*, Clarendon Press, Oxford.

Carter, H. (1976) *The study of urban geography*, 2nd edn, Edward Arnold, London.

Cecchini, A. (1996) "Urban modelling by means of cellular automata: generalised urban automata with the help on-line (AUGH) model", *Environment and Planning B*, vol. 23, pp.721-732.

Clarke, K. C. and Gaydos, L. J. (1998) "Loose-coupling a cellular automaton model and GIS: long-term urban growth prediction for San Francisco and Washington/Baltimore", *International Journal Geographical Information Sciences*, vol. 12, pp.699-714.

Clarke, K. C., Hoppen S. & Gaydos, L. J. (1997) "A self-modifying cellular automaton model of historical urbanization in the San Francisco Bay area", *Environment and Planning B*, vol. 24, pp.247-261.

Couclelis, H. (1997) "From cellular automata to urban models: new principles for model development and implementation", *Environment and Planning B*, vol. 24, pp.165-174.

Couclelis, H. (1989) "Macrostructure and microbehavior in a metropolitan area", *Environment and Planning B*, vol. 16, pp.141-154.

Couclelis, H. (1985) "Cellular worlds: a framework for modelling micro-macro dynamics", *Environment and Planning A*, vol. 17, pp.585-596.

Courtheaux, J. P. (1969) *Classification of economic activities*, IFHP-CRU, The Hague.

Fourastié, J. (1963) *Le Grand Espoir du XX Siécle*, Edition definitive, Gallimard, Paris.

Frost, L. and Dingle, T. (1995) "Sustaining suburbia: An historical perspective on Australia's urban growth", in *Australian cities: strategies and policies for urban Australia in the 1990s*, ed. P. Troy, Cambridge University Press, Melbourne, pp.20-39.

Forster, B. (1993) "Coefficient of variation as a measure of urban spatial attributes using SPOT HRV and Landsat TM data", *International Journal of Remote Sensing*, vol. 14, no. 2, pp.2403-2409.

Forster, B. (1983) "Some urban measurements from Landsat data", *Photogrammetric Engineering and Remote Sensing*, vol. 49, pp.1693-1707.

Fotheringham, A. S. and Wong, D. W. S. (1991) "The modifiable areal unit problem in multivariate statistical analysis", *Environment and Planning A*, vol. 23, pp.1025-1044.

Gardner, M. (1972) "The fantastic combinations of John Conway's new solitaire game 'Life'", *Scientific American*, vol. 233, pp.120-123.

Gryztzell, K. G. (1963) "The demarcation of comparable city areas by means of population density" [translated by Tom Fletcher], *Lund studies in Geography, Series B, Human Geography*, No. 25. Lund: Royal University of Lund, Sweden, Department of Geography.

Gong, P. and Howarth, P. (1990) "The use of structural information for improving land-cover classification accuracies at the rural-urban fringe", *Photogrammetric Engineering and Remote Sensing*, vol. 56, pp.67-73.

Harris, P. and Ventura, S (1995) "The integration of geographic data with remotely sensed imagery to improve classification in an urban area", *Photogrammetric Engineering and Remote Sensing*, vol. 61, no. 8, pp.993-998.

Herbert, D. T. and Thomas, C. J. (1997) *Cities in space: City as place*, 3rd edn, David Fulton Publishers, London.

Hepner, G., Houshmand, B., Kulikov, I. & Bryant, N. (1998) "Investigation of the integration of AVIRIS and IFSAR for urban analysis", *Photogrammetric Engineering and Remote Sensing*, vol. 64, no. 8, pp.813-820.

Itami, R. M. (1994) "Simulating spatial dynamics: cellular automata theory", *Landscape and Urban Planning*, vol. 30, pp.27-47.

Itami, R. M. (1988) "Cellular worlds: models for dynamic conceptions of landscape", *Landscape Architecture*, vol. 78, pp.52-57.

Itami, R. M. and Clark, J. D. (1992) "Spatial dynamic simulations using discrete time and discrete event theory in cell based GIS systems", *Proceedings: 5th International Symposium on Spatial Data Handling*, Charleston, SC., pp.702-712.

Jakobson, L. and Prakash, V. (eds) (1971) *Urbanization and national development*, Beverley Hills, Sage.

Kickert, W. J. M. (1978) *Fuzzy theories on decisions-making: a critical review*, Leiden, Boston.

Kickert, W. J. M. and Mamdani, E. H. (1978) "Analysis of a fuzzy logic controller", *Fuzzy Sets and Systems*, vol. 1, no. 1, pp.29-44.

Kickert, W. J. M. and Van Nanta Lemka, R. H. (1976) "Application of a fuzzy controller in a warm water plant", *Automatica*, vol. 12, no. 4, pp.301-308.

Li, X. and Yeh, A. G. (2000), "Modelling sustainable urban development by the integration of constrained cellular automata and GIS", *International Journal of Geographical Information Systems,* vol. 14, pp.131-152.

Linge, G. J. R. (1965) *The delimitation of urban boundaries for statistical purposes with special reference to Australia*, Australia National University, Canberra.

Mamdani, E. and Assilian, S. (1975) "An experiment in linguistic synthesis with a fuzzy logic controller", *International Journal of Man-Machine Studies*, vol. 7, no. 1, pp.1-13.

McNeill, D. and Freiberger, P. (1994) *Fuzzy logic*, Simon and Schuster, New York.

Mesev, V., Longley, P., Batty, M. & Xie, Y. (1995) "Morphology from imagery: detecting and measuring the density of urban land-use", *Environment and Planning A*, vol. 27, pp.759-780.

Moller-Jensen, L. (1990) "Knowledge based classification of urban area using texture and context information in Landsat TM imagery", *Photogrammetric Engineering and Remote Sensing*, vol. 56, no. 6, pp.899-904.

Openshaw, S. (1984) "The modifiable areal unit problem", *Concepts and Techniques in Modern Geography*, 38, Geo Books, Norwich.

Openshaw, S. and Openshaw, C. (1997) *Artificial intelligence in geography*, John Wiley and Sons, New York.

Phinn, S. R., Stanford, M., Scarth. P., Murray, A. T. & Shyy, P. T., (2001) "Monitoring the composition of urban environments based on the Vegetation-Impervious surface-Soil (VIS) model by sub-pixel analysis techniques", *International Journal of Remote Sensing* (in press).

Pryor, R. J. (1968) "Defining the rural-urban fringe", *Social Forces*, vol. 47, pp.202-215.

Kurtz, R. A. and Eicher, J. B. (1958-59) "Fringe and suburb: a confusion of concepts", *Social Forces*, vol. 37, pp.32-37.

Statistics Canada (1996) *About 1996 Census Tables on the Internet*, Hp. Online. Statistics Canada. Available: http://www.statcan.ca/english/census96/about.html. 20 Jan. 2001.

Takeyama, M. and Couclelis, H. (1997) "Map dynamics: integrating cellular automata and GIS through Geo-Algebra", *International Journal Geographical Information Sciences*, vol. 11, pp.73-91.

Umbers, I. G. and King, P. J. (1980) "An analysis of human decision-making in cement kiln control and the implications for automation", *International Journal of Man-Machine Studies*, vol. 12, no. 1, pp.11-23.

USBC - U.S. Bureau of the Census (1960) *U.S. Census of Population 1960 United States Summary*, PC - IA: xiii.

Wagner, D. F. (1997) "Cellular automata and geographic information systems", *Environment and Planning B*, vol. 24, pp.219-234.

Ward, D. P., Murray, A. T. & Phinn, S. R. (2000) "Monitoring growth in rapidly urbanising areas using remotely sensed data", *Professional Geographer*, vol. 52, no.3, pp.371-386.

Wang, F. and Hall, G. B. (1996) "Fuzzy representation of geographical boundaries in GIS", *International Journal of Geographical Information Systems*, vol. 10, pp.573-590.

Wehrwein, G. S. (1942) "The rural-urban fringe", *Economic Geography*, vol. 18, no. 3, pp.217-228.

Wenstøp, F. (1976) "Deductive verbal models of organizations", *International Journal of Man-Machine Studies*, vol. 8, pp.293-311.

White, R. and Engelen, G. (1997) "Cellular automata as the basis of integrated dynamic regional modelling", *Environment and Planning B*, vol. 24, pp.235-246.

White, R. and Engelen, G. (1994) "Cellular dynamics and GIS: modelling spatial complexity", *Geographical Systems*, vol. 1, pp.237-253.

White, R. and Engelen, G. (1994) "Cellular dynamics and GIS: modelling spatial complexity", *Geographical Systems*, vol. 1, pp.237-253.

White, R. and Engelen, G. (1993) "Cellular automata and fractal urban form: a cellular modelling approach to the evolution of urban land-use", *Environment and Planning A*, vol. 25, pp.1175-1199.

White, R., Engelen, G. & Uljee, I. (1997) "The use of constrained cellular automata for high-resolution modelling of urban land-use dynamics", *Environment and Planning B*, vol. 24, pp.323-343.

Wu, F. (1998a) "An experiment on the generic polycentricity of urban growth in a cellular automatic city", *Environment and Planning B: Planning and Design*, vol. 25, pp.731-752.

Wu, F. (1998b) "Simulating urban encroachment on rural land with fuzzy-logic-controlled cellular automata in a geographical information system", *Journal of Environmental Management*, vol. 53, pp.293-308.

Wu, F. (1998c) "SimLand: a prototype to simulate land conversion through the integrated GIS and CA with AHP-derived transition rules", *International Journal of Geographical Information Science*, vol. 12, pp.63-82.

Wu, F. (1996) "A linguistic cellular automata simulation approach for sustainable land development in a fast growing region", *Computers, Environment, and Urban Systems*, vol. 20, pp.367-387.

Wu, F. and Webster, C. J. (2000) "Simulating artificial cities in a GIS environment: urban growth under alternative regulation regimes", *International Journal of Geographical Information Science*, vol. 14, no. 7, pp.625-648.

Wu, F. and Webster. C. J. (1998) "Simulation of land development through the integration of cellular automata and multicriteria evaluation", *Environment and Planning B: Planning and Design*, vol. 25, pp.103-126.

Wolfram, S. (1984) "Cellular automata as models of complexity", *Nature*, vol. 311, pp.419-424.

Woodcock, C. E. and Gopal, S. (2000) "Fuzzy set theory and thematic maps: accuracy assessment and area estimation", *International Journal of Geographical Information Science*, vol. 14, no. 2, pp.153-172.

Zadeh, L. A. (1973) "The concept of a linguistic variable and its application to approximate reasoning", *Memorandum ERL-M 411*, Berkeley, October 1973.

Zadeh, L. A. (1971) "Towards a theory of fuzzy systems", in *Aspects of Network and Systems Theory*, eds R. E. Kalman & N. DeClaris, Holt Rinehart, Winston.

Zadeh, L. A. (1965) "Fuzzy sets", *Information and Control*, vol. 8, pp.335-353.

Zadeh, L. A. (1962) "From circuit theory to systems theory", *IRE Proceedings*, vol. 50, pp.856-865.