

# **Investigation of Spatial Interpolation Methods to Determine the Minimum Error of Estimation: Case Study, Temperature and Evaporation**

**Mohammad Hossein Mahdian  
Ebrahim Hosseini  
Mahmoud Matin**

**Soil Conservation Watershed Management Research Center, P.O. Box: 13445-1136, Tehran, Iran**

## **ABSTRACT**

One of the basic requirements for researching on agriculture and natural resources is climatic data. Most of basins in Iran have not any weather station or also because of insufficient data and short observed records, then, the estimation of data for larger area is difficult. Moreover, most times one or more methods is selected regardless of considering other methods. The application of interpolation methods for estimation meteorological data has not been widely used in Iran. In the present research, three following methods have been used: Kriging, weighted moving average (WMA), and thin plate smoothing splines (TPSS). The results of this research show that TPSS is the most appropriate for estimation of monthly and annual evaporation. Also, The TPSS method shows the most precise results for estimation of both monthly and annual air temperature. In addition, the effect of climate scheme on the accuracy of temperature and evaporation was investigated. Based on the results, the climate scheme a positive effect on temperature, but no effect on evaporation was observed.

**KEYWORDS:** Temperature,Evaporation, Spatial Interpolation, Rainfall, Kriging, TPSS, WMA.

## **INTRODUCTION**

Evaporation and temperature have an important role in the study of soil and water management. These variables control the parameters such as time of germination, water requirements and cultural operations. Therefore, measuring and determination of these variables is vital in different domains. Various methods to spatially interpolate sparse observational data have been used including partitioning into regions, Thiesen method, nearest points, moving averages, kriging and thin plate smoothing splines. Of these methods, statistical interpolation techniques appear to be best suited to the task. This methodology has been used for environmental data in different diceplin such as soil salinity, pH, hydraulic conductivity, geohydrology and so on. Hosseini et al. (1993) compared several interpolators to estimate hydraulic conductivity to create iso-contoure in south west of Iran. Laslett et al. (1987) applied different techniques to estimate soil-surface acidity, they reported that Kriging with Laplace interpolators has a superior results. The technique includes the method of thin plate smoothing splines (Wahba, 1980, Hutchinson 1991) and kriging (Delfiner and Delhomme 1975, Cressie 1991) has given the more precise results than other methods. These techniques model spatial distribution as a function of observational data across a region without prior knowledge of the distribution or its underlying physical causes. Hutchinson and Gessler (1994) and Hutchinson (1993) have extensively investigated spliane and kriging for climatic data. Voltz and Goulard (1994) compared the performance of three interpolation

methods (nearest neighbor, ordinary kriging, and cokriging) for soil moisture retention curves and found that ordinary kriging and cokriging were the most accurate. Studies by Hosseini et al. (1994) give an overview of the theoretical and experimental of geostatistical methods and thin plate smoothing splines. There is a practical difference between these methods. Kriging requires the spatial covariance function or variogram to be estimated first, and is critical to the process. Thin plate smoothing splines, on the other hand, require the estimation of a smoothing parameter that determines an optimal balance between fidelity to the data and smoothness of the fitted spline function.

Therefore the objective of this study is to evaluate the applicability of different interpolation methods in estimating monthly and annual temperature and evaporation in the center of Iran.

## **MATERIALS AND METHODS**

### **Description of the study area**

Study areas are located in arid and semiarid regions of the center of Iran. The study area is presented in (Fig. 1). This region covers about 82.4 million ha that is approximately half of Iran's soil. This area includes seven watersheds that received less than 50% of total annual rainfall of the country. However, 55% of total groundwater is extracted from these areas. For that reason, the antian civilizations are established in this area. Today, the big cities such as Tehran, Karaj, Esfahan, Shiraz, Semnan, Yazed, Kerman, and Hamadan are located in this area. The region has a longitude between 50° 0' 0" and 62° 49' 48" and latitude between 25° 16' 48" and 43° 0' 0". It has a high variation in elevation, which varied between 353 and 2800 m.

### **Meteorological data**

The 80 and 222 stations are used for evaporation and temperature, respectively. Figures 2 and 3 illustrated the location of meteorological stations by which evaporation and temperature data has been measured. As we can see, the density of weather stations in west of study areas is more than the rest of whole areas. This distribution is normally based on human activities and natural resources production.

## **INTERPOLATION METHODS**

The several interpolator methods for estimating a variable that changes in space and time are considered. The main differences of these techniques are estimation of weights that depends of radius of search on selected number of neighbors. The methods are used in this study are abbreviated in Table 1. Local interpolators such as kriging and moving average, and inverse distance methods use the following equation:

$$Z^* = \sum_{I=1}^n I_I \times Z_I \quad (1)$$

Where:

$Z^*$  = estimated value,  
 $Z_i$  = observed value,  
 $n$  = number of observation,  
 $i$  = index for the number of stations.

Table 1. Interpolation methods used in this study.

<b>Abbreviation</b>	<b>Method</b>
<b>WMA</b>	Weighted Moving Average
WMA-1	a=1
WMA-2	a=2
WMA-3	a=3
WMA-4	a=4
<b>OK</b>	Ordinary Kriging
L-kriging	With logarithme of data
Kriging	Kriging only
<b>TPSS</b>	Thin plate smoothing splines
TPSS-2	a=2
TPSS-3	a=3
	With co variable
TPSS-CO2	a=2
TPSS-CO3	a=3

### EVALUATION CRITERIA

The performance of estimation methods was evaluated by cross validation. In this procedure, an observed value is temporarily discard from the sample data set, and one estimated value at that location is determined using the other sample points. This results in a series of observed and estimated values that can be used to assess the validity of the interpolation method. In this study, estimated and observed values were compared using two criteria: the mean bias error (MBE) and the mean absolute error (MAE):

$$MBE = \frac{\sum_{i=1}^n (E_s - E_o)}{n} \tag{2}$$

$$MAE = \frac{\sum_{i=1}^n |E_s - E_o|}{n} \quad (3)$$

Where:

$n$  = number of values,

$E_s$  = estimated value,

$E_o$  = observed value,

$i$  = index for the number of data.

$MBE$  = Mean bias error

$MAE$  = Mean absolute error

## RESULTS AND DISCUSSION

Statistical analysis of evaporation data shows that the average of elevation in the study area is 2511 m and varied from 1313 to 4692 m. The average of temperature in this region is about 14.2 °C with a variation from 7.9 to 25.2 °C. Coefficient of variation (C.V) for monthly evaporation ranges from 1.1 °C for August to 148.5 °C for January. For temperature, the C.V varies between 11.9 °C for August to 152 °C for January. The results show that the coefficient of variation for both variables is high, for this reason, elevation as an important parameter affecting these variable, has been considered as a covariable in this study. According to information of evaporation station, the minimum elevation was 353 m and the maximum was 2310 m above sea level with average of 1543 m and standard deviation of 438 m. The average of elevation of stations used for temperature is 1564 m that varied from 353 to 2800 m. The relationship between evaporation (E), temperature (T) with elevation (H) are presented in the following equations:

$$E = -0.3299 \times H + 3021 \quad R^2 = 0.0468 \quad (4)$$

$$T = -0.0053 \times H + 22.247 \quad R^2 = 0.5324 \quad (5)$$

These equations show that the correlation between both variables and elevation is not high especially for evaporation. This can be explained by the fact that study area is very vast. Therefore, the correlation between these two variables was calculated for sub-watersheds to improve the correlation. The results are presented in Table 2 and 3 for evaporation and temperature verses elevation, respectively.

### Monthly Evaporation

Statistical analysis is shown that evaporation data for all months obeys normal distribution. For instance, distribution of evaporation data for January is illustrated in Fig 4. To analysis the spatial correlation of monthly evaporation, experimental variograms are computed for

different months. General shape of variogram shows a high spatial dependency of evaporation data and modeled by Gaussian model. Nugget effect ( $C_0$ ) ranges from 27 to 6566 mm and sill (C) ranges from 405 to 11172 mm with range of influence varied from 39 to 64 km for different months. The experimental variogram with fitted model is presented for the month of January in Fig 5.

### Monthly Temperature

The same procedure carried out for monthly temperature as done for evaporation. Distribution for temperature look likes normal for all of months. Fig 6. represents as an example of monthly temperature for January. The results of evaluation of temperature for different methods are presented in Table 4. Variography of temperature data done by computing experimental variogram and represents a good spatial correlation that can be seen stronger than evaporation. Structural analysis shows

Table 2. Correlation between evaporation and elevation in the sub-watersheds.

Watershed	Number of Stations	Equation	Correlation ( $R^2$ )
Ghom	39	$Y = -0.027X + 2691$	0.21
Dasht-Kavier	9	$Y = -1.480X + 4552$	0.55
Gavkhoni-Esfahan	13	$Y = -0.780X + 2770$	0.22
Yazed - Ardestan	5	$Y = 0.470 X + 2341$	0.57
Kavier-Lout	6	$Y = -0.459X + 3973$	0.53
Maharlo	8	$Y = 0.4900X + 1391$	0.26
Jazmorian	0	-	

Table 3. Correlation between temperature and elevation in the sub-watersheds.

Sub-watershed	Number of Stations	Equation	Correlation ( $R^2$ )
Ghom	43	$Y = -0.0025X + 21.3$	0.77
Dasht-Kavier	35	$Y = -0.0070X + 2413$	0.77
Gavkhoni- Esfahan	40	$Y = -0.0060X + 27.37$	0.85
Yazed - Ardestan	21	$Y = 0.0051 X + 24.3$	0.79
Kavier-Lout	18	$Y = -0.0059X + 25.3$	0.84
Maharlo	14	$Y = 0.0022X + 19.0$	0.46
Jazmorian	0	-	

that more spherical or Gaussian model are fitted to experimental variogram. In general, nugget effect ( $C_0$ ) ranges from 1.83 to 3.64 °C and sill (C) ranges from 5.82 to 12.18 °C. Range of influence varies from 146 to 245 km. The experimental variogram with fitted model of January is pictured in Fig 7. In this study we were concerned about WMA method with

regarding to number of stations that have be used to estimation process. In this study we found that, at least 10 stations are needed to have more precision (MAE) and low error of estimation (MBE). The results of evaluation of interpolator methods are presented in Table 4. Based on this investigation, TPSS-CO2 and L-Kriging give more precision than other methods. However, WMA-1 and TPSS-3 methods represent less accuracy of estimation.

Table 4. Results of evaluation of different methods in estimating monthly evaporation and temperature.

Methods	Temperature		Evaporation	
	MAE	MBE	MAE	MBE
<b>WMA-1</b>	1.5	0.036	37.3	-4
<b>WMA-2</b>	1.42	0.051	37.3	-6.4
<b>WMA-3</b>	1.42	-0.048	38.4	-7.8
<b>WMA-4</b>	1.44	0.047	38.8	-7.8
<b>L-kriging</b>	1.04	1.00	37.1	-8.0
<b>Kriging</b>	1.51	0.008	36.8	-3.2
<b>TPSS-2</b>	1.4	0.1	37.2	-6.8
<b>TPSS-3</b>	1.5	0.0	36.3	-6.5
<b>TPSS-CO2</b>	1.03	0.04	36.3	-6.7
<b>TPSS-CO3</b>	1.09	0.02	36.2	-7.1

## Annual Temperature and Evaporation

Histogram of annual temperature and evaporation are presented in the figure 9 and 10, respectively. The results of evaluation of different interpolation methods for estimating of annual temperature and evaporation are shown in Table 5. Based on the results of this table, two methods as L-Kriging and TPSS-CO2 give a reasonable degree of precision (MAE =1.06 °C) for annual temperature in studies area. The MBE was 1.0 and 0.02 °C for L-Kriging and TPSS-CO2, respectively. But, for estimation of evaporation, the TPSS-CO3 method come with precision of 378.5 mm with corresponding MBE = -84.3 mm. Another interesting point that can be concluded from Table 5 is that all methods overestimated temperature but had underestimation results for evaporation.

Table 5. Results of evaluation of different methods in estimating yearly evaporation and temperature.

Methods	Temperature		Evaporation	
	MAE	MBE	MAE	MBE
<b>WMA-1</b>	1.47	0.04	399.0	-50.0
<b>WMA-2</b>	1.38	0.05	390.0	-79.0
<b>WMA-3</b>	1.34	0.04	403.0	-93.0
<b>WMA-4</b>	1.34	0.04	417.0	-99.0
<b>L-kriging</b>	1.06	1.00	383.0	-81.0
<b>Kriging</b>	1.51	0.008	387.0	-37.0
<b>TPSS-2</b>	1.12	0.04	411.6	-75.9
<b>TPSS-3</b>	1.12	0.04	384.5	-77.1
<b>TPSS-CO2</b>	1.06	0.02	395.5	-81.4
<b>TPSS-CO3</b>	1.12	0.04	378.5	-84.3

## Climatic Investigation

With regarding to have a large area with different microclimate, and, based on precision of interpolators and lower MBE, study area has been divided in three-sub area of climatic oriented. It is important to explain that the number of stations is limited to each sub area. Then, three class of climatic sub area are recognized based on Guassian classification as: sever arid, semi-arid with cold region and Mediterranean. Figure 8 shows the sub climatic areas. Therefore, the selected interpolator technique as kriging was applied to estimate evaporation and temperature. The results of the evaluation for three sub- climatic areas are presented in Table 6.

Table 6. Results of the appropriated methods for evaporation and temperature comparing to total area.

Variable		Sever arid and Semi-Arid		Semi-arid with cold region		Mediterranean rgime		Total
		MAE	MBE	MAE	MBE	MAE	MBE	MAE
Evaporation (mm)	value	68.4	-3.5	40.9	-4.5	33.9	-0.4	
	%	27.8		20.2		22.5		20.4
Temperature (°C)	value	1.04	-0.42	1.05	0.05	1.07	-0.58	
	%	6.7		11.4		10.8		16.0

As we can see in Table 6, within sub areas only the semi-arid with cold regime gives 0.2 percent more precision. In other sub areas, the precision decreased from 2.1 to 7.4. This results for arid climate area maybe caused by a few number of stations that exist in this area.

For temperature, the accuracy of estimators increased for sub areas compare of total area. Maybe these results related to density of weather stations that distributed in sub areas.

## **CONCLUSIONS**

Different interpolation methods were evaluated for estimating evaporation and temperature in center region of Iran. Based on the results of this study the following conclusions can be obtained:

- 1- The range of influence is 89 and 181 km for annual evaporation and temperature, respectively.
- 2- For temperature, the range of influence in cold months is bigger than hot months, that, its average values are 195 and 165 km in cold and hot months, respectively. Contrary to temperature, the average of range of influences are 45 and 133 km in cold and hot months, respectively.
- 3- The TPSS method was recognized as the most precise method for estimating monthly and annual temperature and evaporation.
- 4- By dividing study region in sub watershed, precision of estimation has been improved considerably for temperature but no suitable changes was obtained for evaporation.

## **REFERENCES**

- Cressies, N. 1991. Statistics for spatial data, John Wiley and Sons, New York, N.Y.
- Delfiner P. and J.P. Delhomme 1975. Optimum interpolation by kriging. In: J.D.Davis and M.J.McCullagh (eds), Display and Analysis of Spatial Data. New York, John Wiley and Sons, pp 96-114.
- Hosseini, e. j. Gallichand and J. Caron. 1993. Comparison of several interpolators for smoothing hydraulic conductivity data in South West Iran. Transactions of the ASAE 37(6):1799-1807.
- Hosseini, E., J. Gallichand and D. Marcotte. 1994. Theoretical and experimental performance of spatial interpolation methods for soil salinity analysis. Transactions of the ASAE 36(7): 1799-1807.



Hutchinson M.F. 1991. Continent-wide data assimilation using thin plate smoothing splines. In: J.D.Jasper (ed), Data Assimilation Systems. BMRC Research Report No.27, Melbourne: Bureau of Meteorology, pp 104-113.

Hutchinson M.F. 1993. On thin plate splines and kriging. In: M.E. Tarter and M.D. Lock (eds), Computing Science and Statistics, Vol. 25, Interface Foundation Of North America, University of California, Berkeley, pp 104-113.

Hutchinson M.F. and Gessler P.E. 1994. Splines - more than just a smooth interpolator. *Geoderma* 62: 45-67.

Laslett G.M., A.B. Mcbratney, P.I. phal and M.F. Hutchinson. 1987. Comparison of several spatial prediction methods for soil pH. *Journal od Soil Science* 38:325-341.

Voltz, M. and M. Goulard. 1994. Spatial interpolation of soil moisture retention curves. *Geoderma* 62:109-123.

Wahba, G. 1980. Splines models for observational data, CBMS\_NSF regional conference series in applied mathematics 59. Philadelphia, Pa.:SIAM.



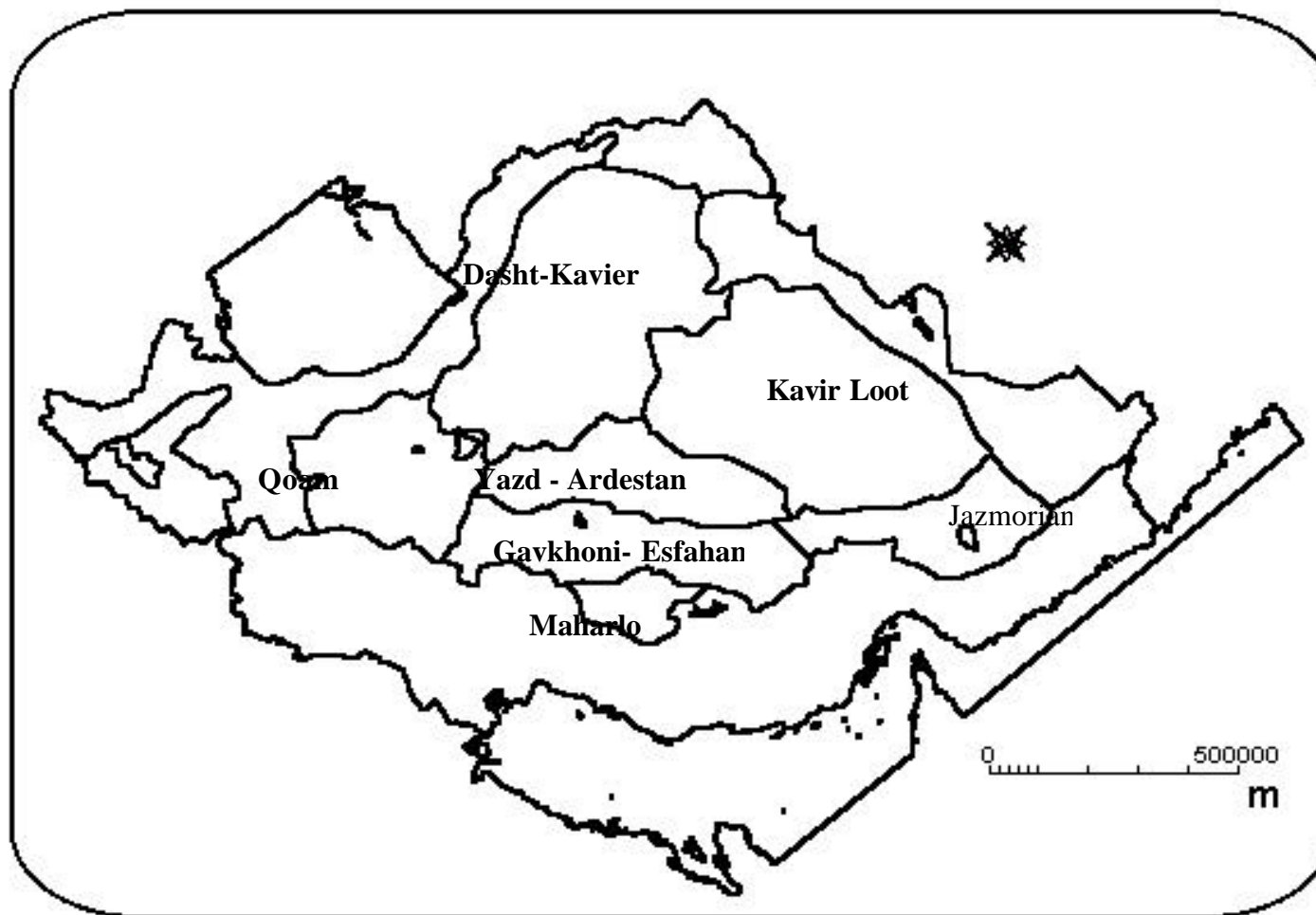


Fig. 1. The study area.

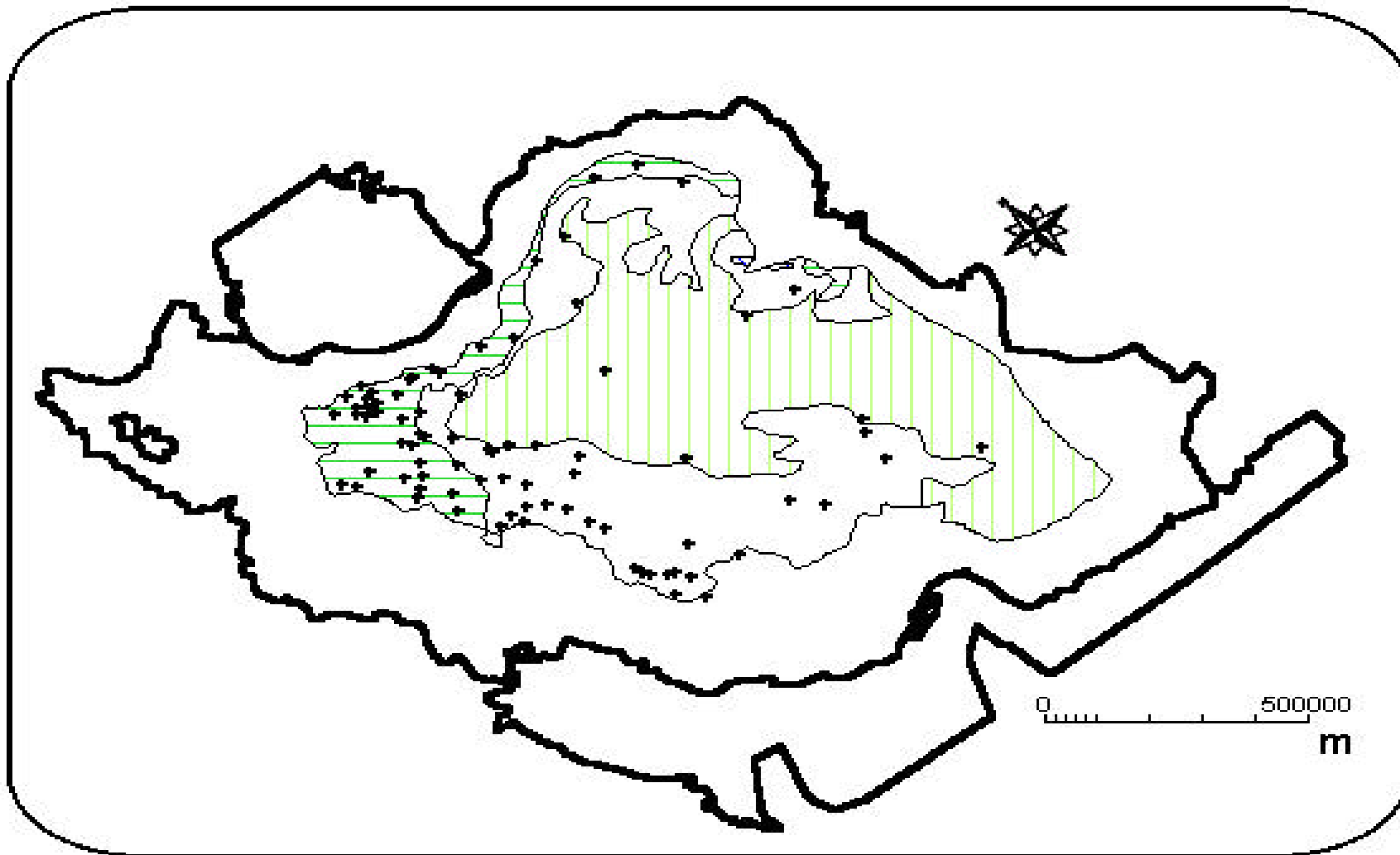


Fig 2. Location of meteorological stations measured evaporation.

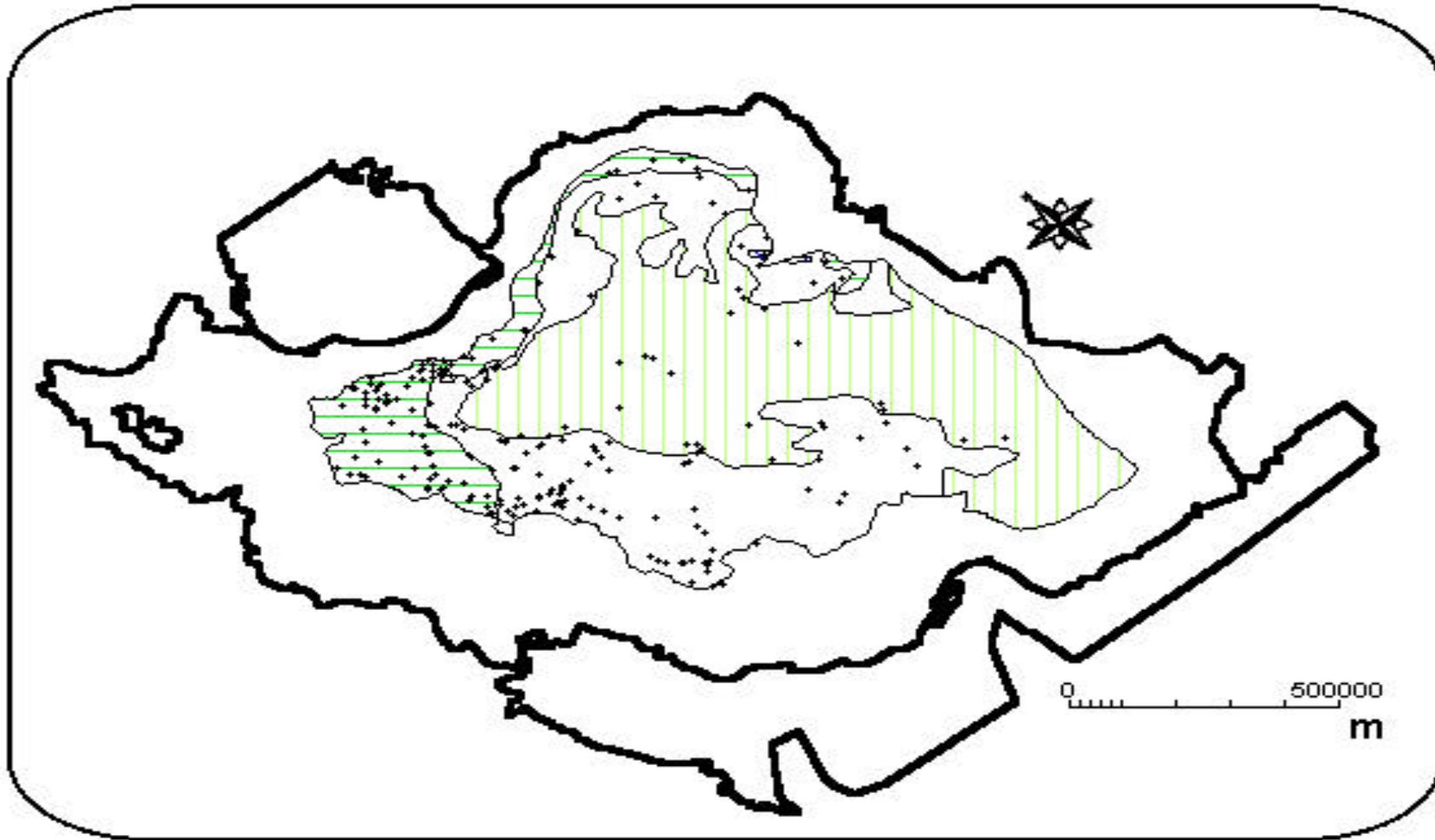


Fig. 3. Location of meteorological stations measured temperature in the center of Iran.

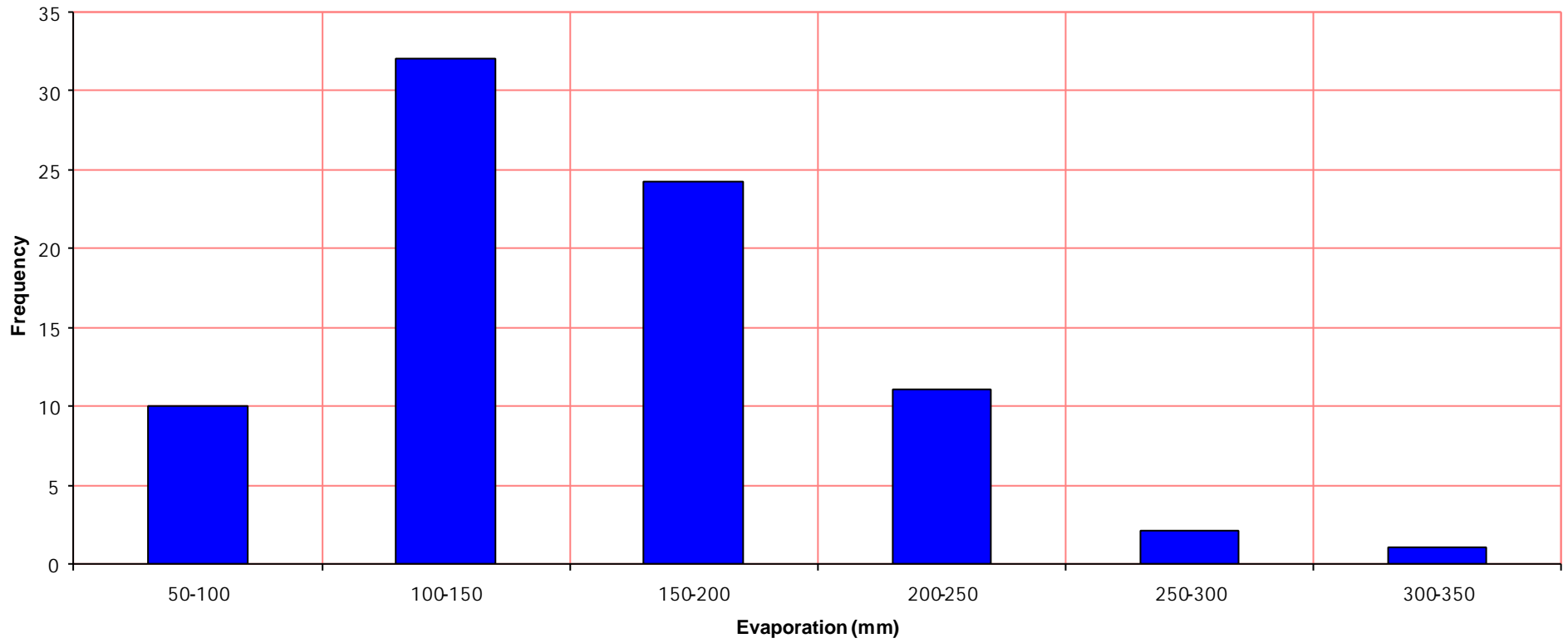
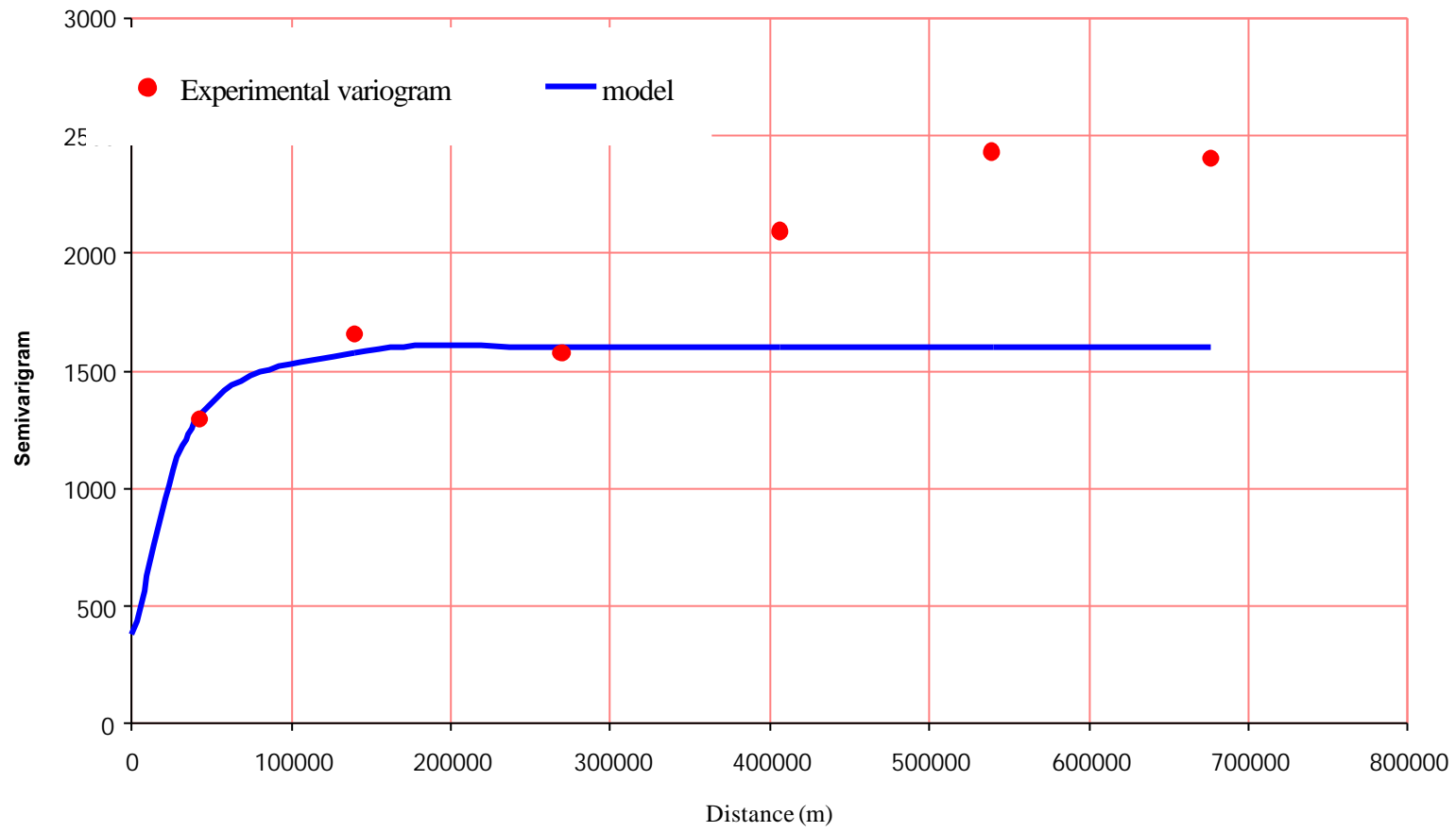


Fig. 4. Histogram of evaporation for the month of Janaur



**Fig. 5. Model and experimental semivariogram of evaporation for the month of janau**

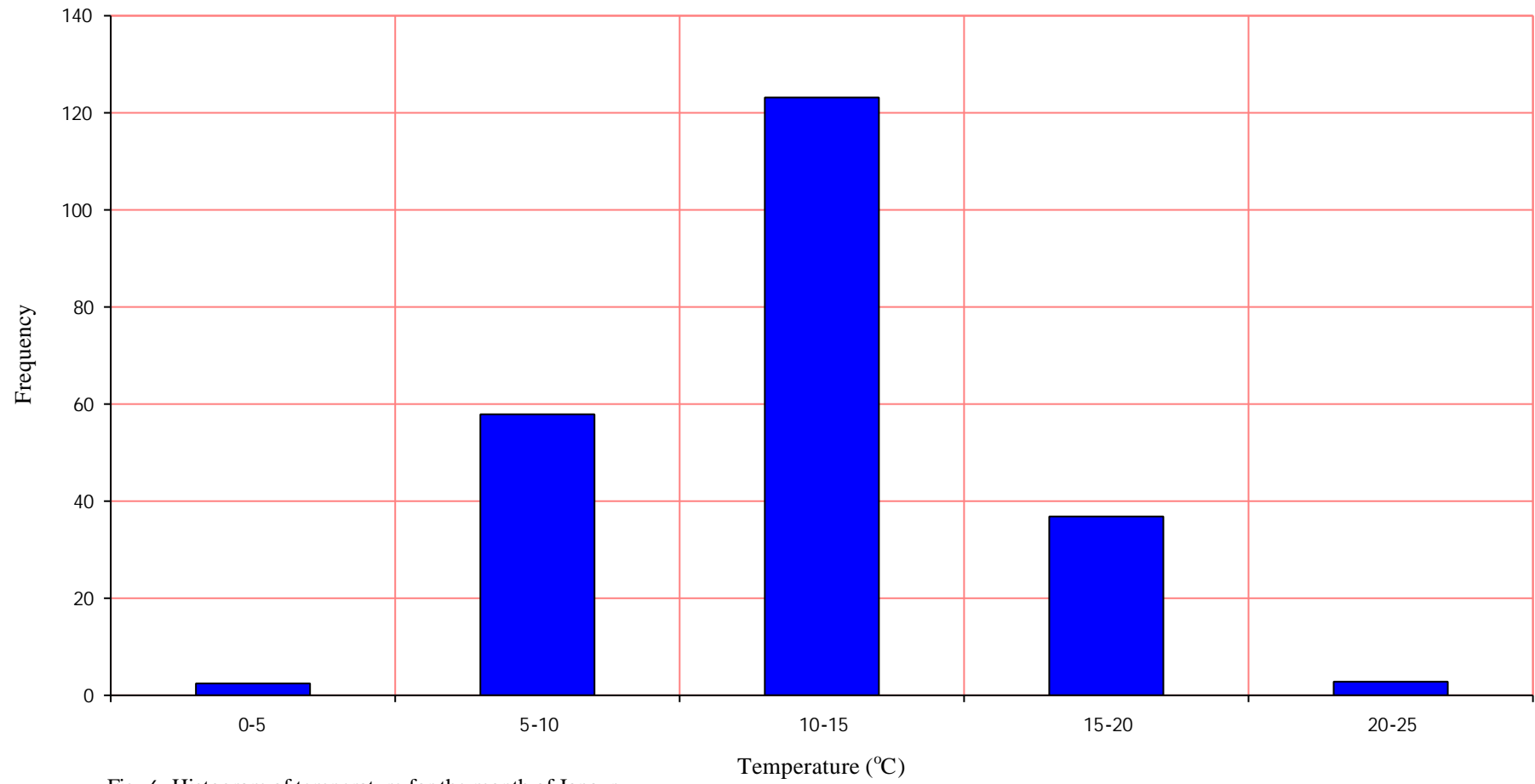


Fig. 6. Histogram of temperature for the month of January .



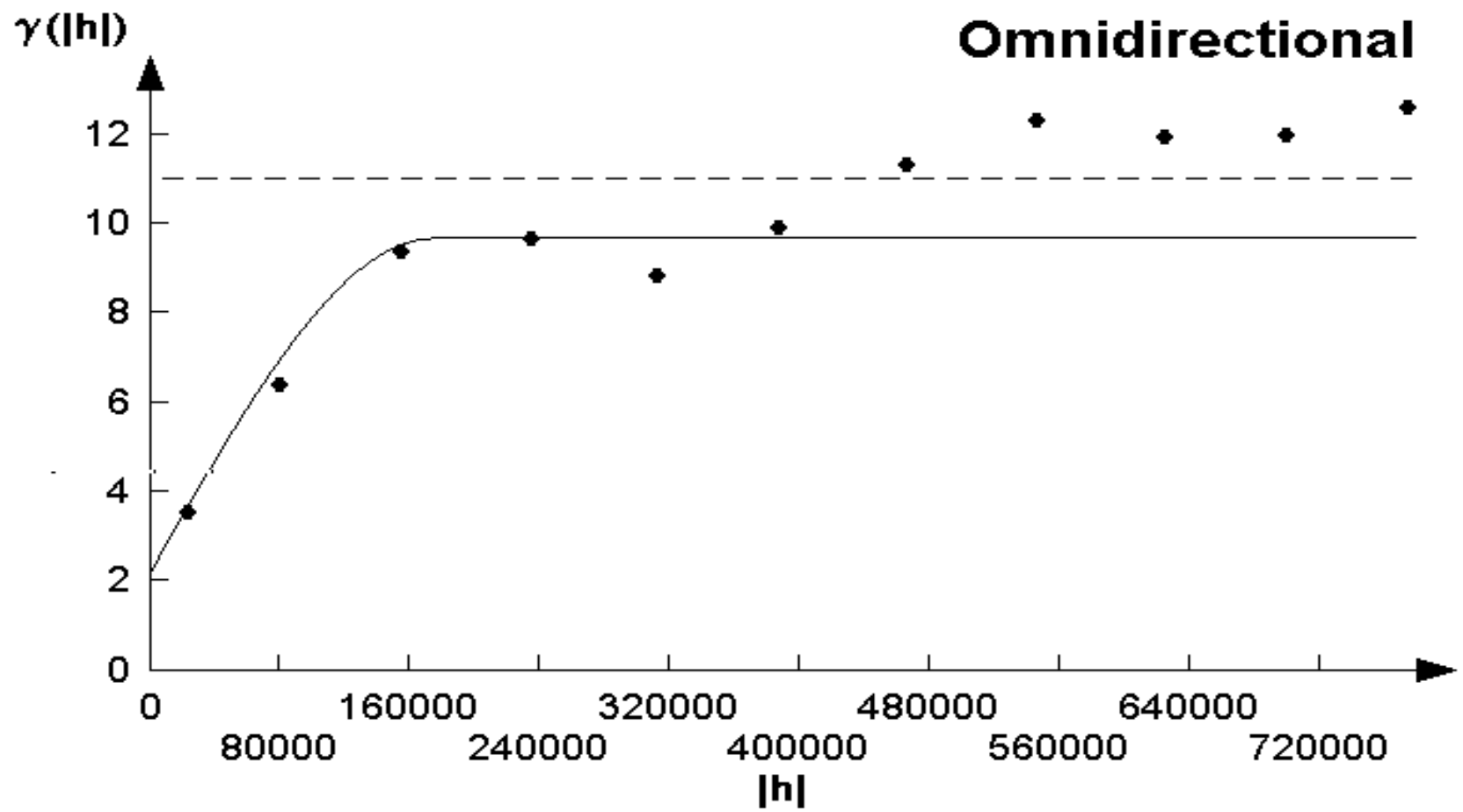


Fig. 7. Model and experimental semivariogram for temperature for the month of January.

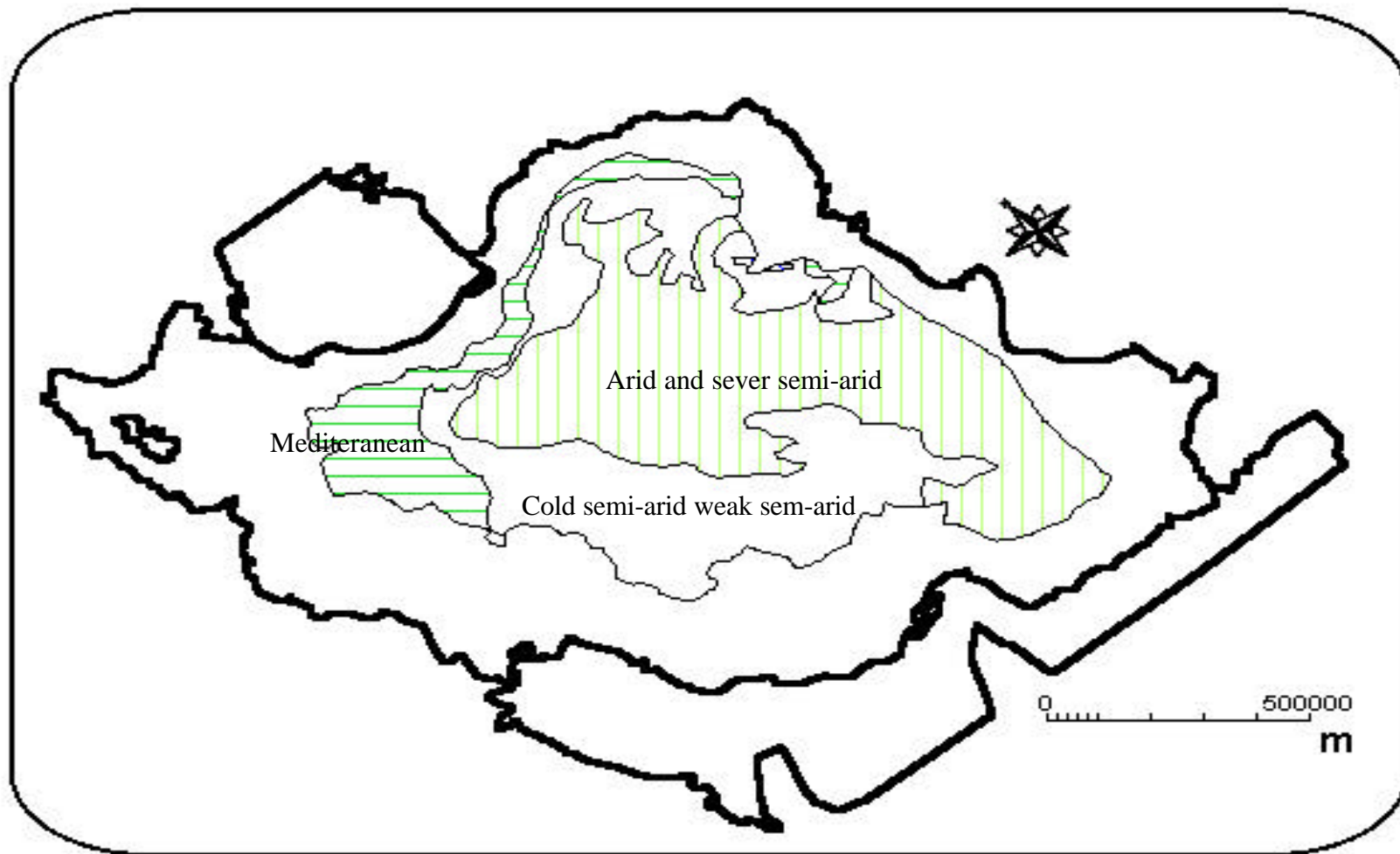


Fig. 8. Sub-climate defined in the center of Iran



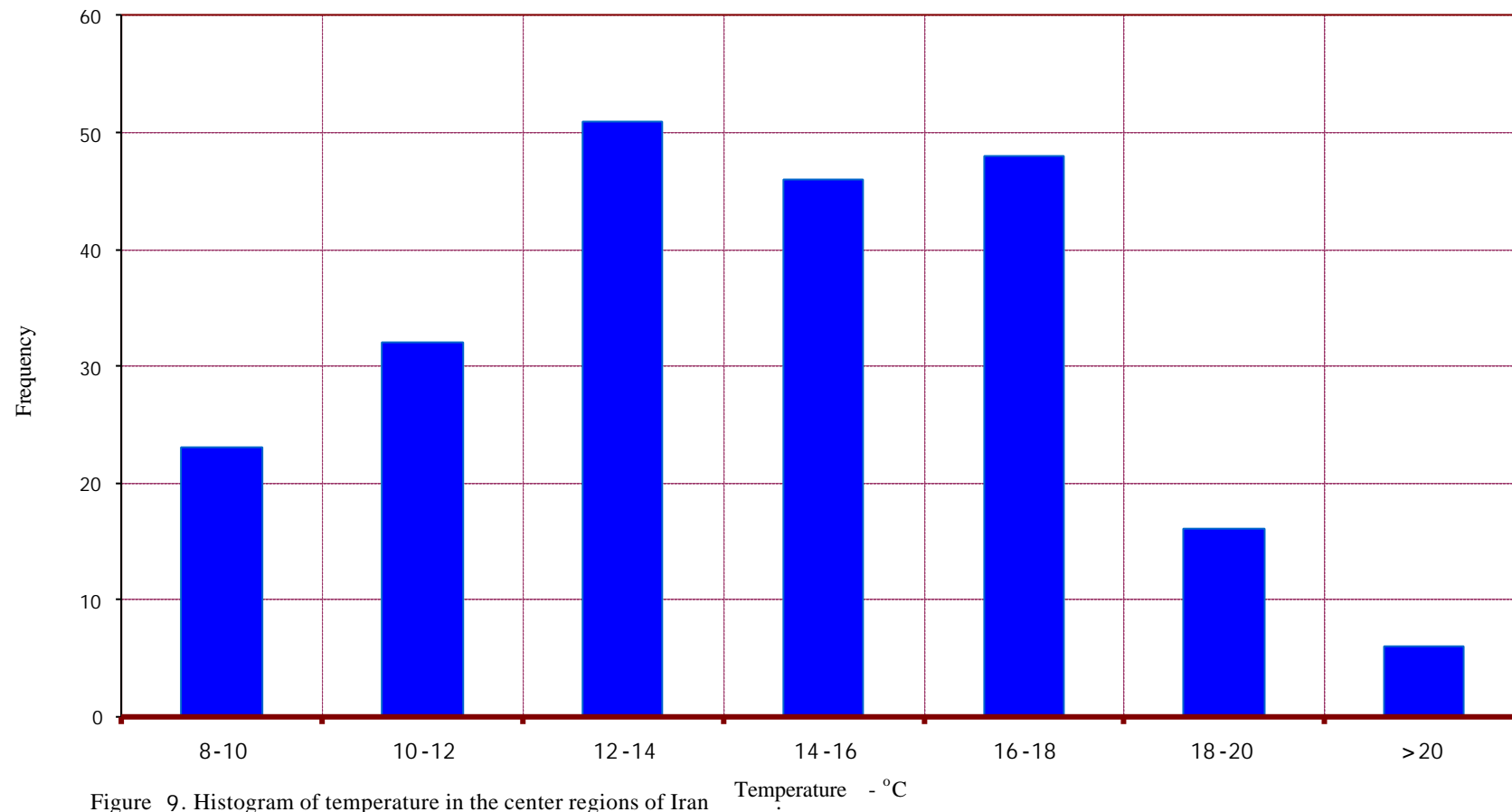
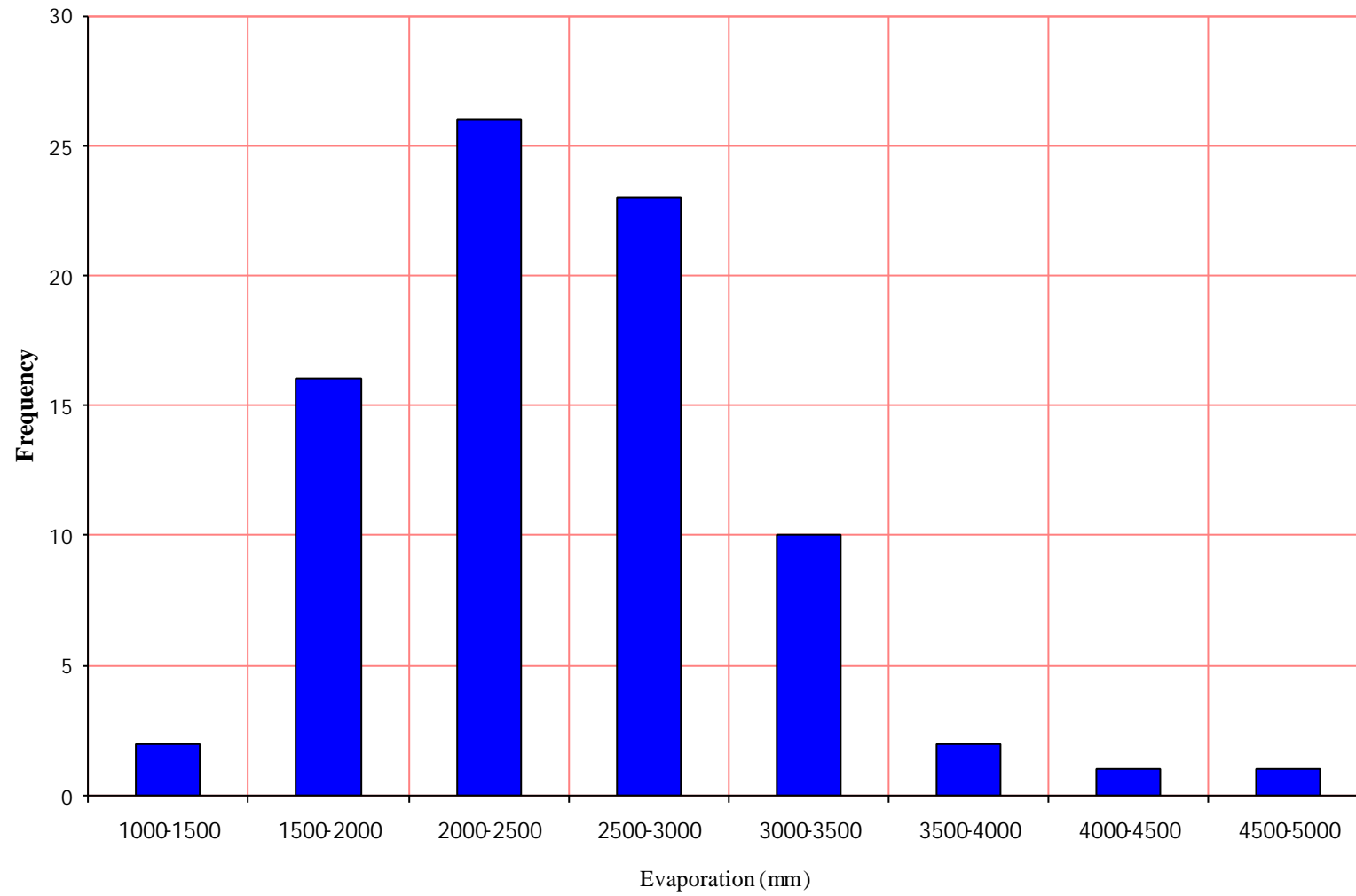


Figure 9. Histogram of temperature in the center regions of Iran Temperature - °C



**Fig 10** Histogram of yearly evaporation for the center region of Iran .