Variable Resolution Spatial Interpolation Using The Simple Recursive Point Voronoi Diagram

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Biography

Barry Boots is Professor of Geography and Environmental Studies at Wilfrid Laurier University. His main interests are measuring spatial structure and spatial relationships, spatial statistics, and Voronoi diagrams.

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The basic Voronoi concept involves tessellating an m-dimensional space with respect to a finite set of objects by assigning all locations in the space to the closest member of the object set. This concept can be operationalised in a variety of ways by considering different methods for determining "closeness", subsets of objects rather than individual objects as generators, moving objects, and different types of spaces, as well as various combinations of these. This versatility has resulted in Voronoi constructions being used increasingly in a wide range of disciplines for spatial data manipulation, spatial interpolation, modelling spatial structures and spatial processes, pattern analysis, and locational optimization (for a review see Okabe *et al.*, 2000).

The Voronoi concept may also be applied recursively by tessellating the space with respect to a given set of generators and then repeating the construction every time with a new generator set consisting of objects selected from the previous generator set plus features of the current tessellation. More formally, consider a finite set of n distinct generators, G. First, construct the Voronoi diagram $V(G)$ of G. Next, extract a set of features Q from $V(G)$ and create a new set of generators G' which comprises of Q plus selected members of G. Then construct the Voronoi diagram $V(G')$ of G'. This step is then repeated a number of times. At each step, the number of generators that are retained may range from none to all. Boots and Shiode (2002) show that such recursive constructions provide an integrative conceptual framework for a number of disparate procedures in spatial analysis and modelling.

In this paper, we consider a specific form of recursive Voronoi diagram, the simple recursive point Voronoi diagram (SRPVD), in which the initial set of generators consists of n distinct points $G_{(0)} = \{g_{(0)1}, g_{(0)2}, \ldots, g_{(0)n}\}$. The construction of this diagram involves the following steps:

0. Define the initial set of generators $G_{(0)}$;
1. Generate the ordinary Voronoi diagram $V(G_{(0)})$ of $G_{(0)}$;
2. Extract all $m(0)$ Voronoi vertices $Q_{(0)} = \{q_{(0)1}, q_{(0)2}, \ldots, q_{(0)m(0)}\}$ of $V(G_{(0)})$;
3. Create a new set of generator points $G_{(1)} = G_{(0)} + Q_{(0)}$;
4. Repeat steps 1 through 3.

We call the result of the k^{th} construction, $V(G_{(k)})$, the k^{th} generation of the simple recursive point Voronoi diagram. Similarly, we call its generator set $G_{(k)}$ and its vertices $Q_{(k)}$, the k^{th} generation of the point set and the k^{th} generation of the vertices, respectively. The results of applying these steps for five recursions to an initial generator set consisting of five points are shown in Figure 1. Note that the Voronoi polygons at generation k are either entirely contained within a Voronoi polygon at generation (k-1) (i.e., $V(G_{(k)\,i}) \phi V(G_{(k\text{-}1)\,i})$ or are composed of pieces of three polygons from generation (k-1), i.e., $V(G_{(k)\,i}) = (V(G_{(k)\,i}) \cap V(G_{(k\text{-}1)l})) \cup (V(G_{(k)\,i}) \cap V(G_{(k\text{-}1)m})) \cup (V(G_{(k)\,i}) \cap V(G_{(k\text{-}1)n}))$.

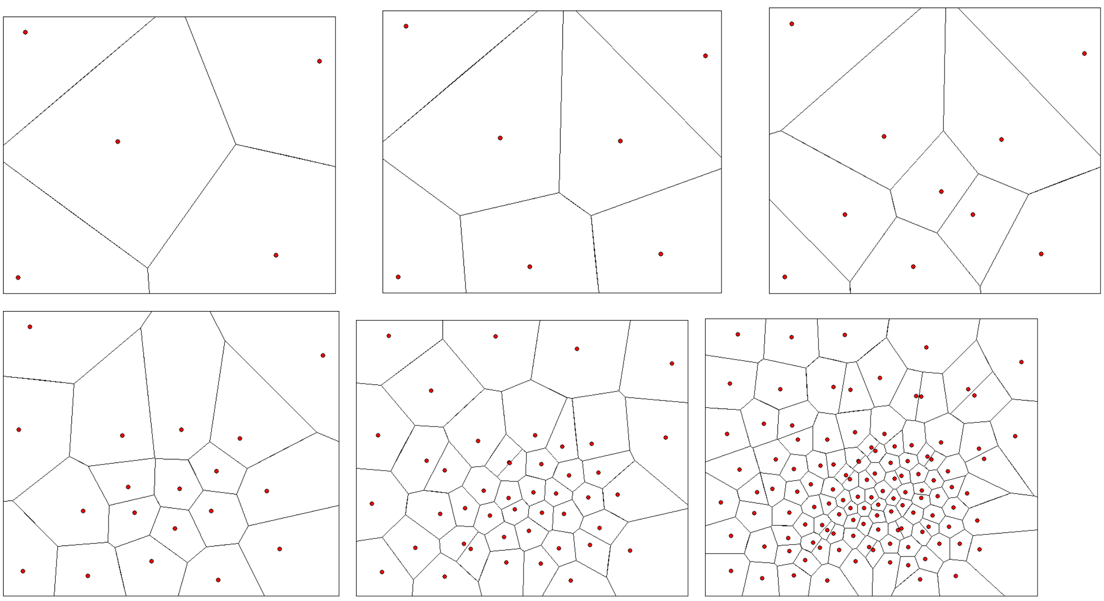

Figure 1. The first five recursions of a Simple Recursive Point Voronoi Diagram

Since the SRPVD provides a means of progressively increasing the density of an initial point set, we propose that it provides a basis for interpolating complex surfaces from comparatively sparse sample sets of both quantitative and categorical data. Two factors motivate this activity. First, notwithstanding advances in remote sensing and GPS data collection methods, in some instances it can still be impractical or too costly to collect appropriately detailed data samples. Second, while the capacity of GIS to manage large volumes of data has increased markedly in recent years, substantial data storage and processing performance advantages can be realised with data compression or simplification methods.

A procedure to produce topographic surfaces that are progressively more complex and of variable spatial resolution can be defined as follows.

Let $G_{(0)}$ be an initial set of generators each with an associated data value and label the set of data values $D_{(0)}$.
Generate $V(G_{(0)})$.
Extract the set $Q_{(0)}$ consisting of the $m(0)$ vertices of $V(G_{(0)})$.
Extrapolate data values for $Q_{(0)}$ using Sibson's natural neighbour interpolation (Sibson, 1981) and $D_{(0)}$.
Label extrapolated values $D_{(1)}$.
If desired, create surface representation from $D_{(0)} + D_{(1)}$.
Let $G_{(1)} = G_{(0)} + Q_{(0)}$.
Generate $V(G_{(1)})$.
Extract the set $Q_{(1)}$ consisting of the $m(1)$ vertices of $V(G_{(1)})$.
Extrapolate data values for $Q_{(1)}$ using Sibson's natural neighbour interpolation and $D_{(0)} + D_{(1)}$.
Label extrapolated values $D_{(2)}$.
If desired, create surface representation from $D_{(0)} + D_{(1)} + D_{(2)}$.

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Let $G_{(k)} = G_{(k-1)} + Q_{(k-1)}$.
Generate $V(G_{(k)})$.
Extract the set $Q_{(k)}$ consisting of the $m(k)$ vertices of $V(G_{(k)})$.
Extrapolate data values for $Q(k)$ using Sibson's natural neighbour interpolation and $D_{(0)} + D_{(1)} + \ldots + D_{(k)}$.
Label extrapolated values $D_{(k+1)}$.
If desired, create surface representation from $D_{(0)} + D_{(1)} + \ldots + D_{(k+1)}$.

For generators in general quadratic position, each interpolated value in $D_{(k+1)}$ will be interpolated from three values in $D_{(0)} + D_{(1)} + \ldots + D_{(k)}$. If $D_{(k+1)\,i}$ is the interpolated value at $Q_{(k)i}$ and $V(Q_{(k)i})$ is the Voronoi polygon of $Q_{(k)i}$, and $*V(Q_{(k)i})*$ is the area of $V(Q_{(k)i})$, then

$$D_{(k+1)i} = \sum_{p=1}^{3} \frac{\left|V\left(Q_{(k)i}\right) \cap V\left(Q_{(k-1)p}\right)\right|}{\left|V\left(Q_{(k)i}\right)\right|} D_{(k)\,p} \qquad [1]$$

Note that

$$\left|V\left(Q_{(k)i}\right)\right| = \sum_{p=1}^{3} \left|V\left(Q_{(k)i}\right) \cap V\left(Q_{(k-1)p}\right)\right|$$

If $D_{(0)}$ consists of quantitative values, a single value can be interpolated at each vertex. If $D_{(0)}$ consists of categorical values, a vector of Fuzzy Membership Values (FMVs) for classes can be interpolated at each vertex (Lowell, 1994) and equation [1] will need to be adjusted accordingly. FMV surfaces can then be generated for each class. Alternatively, at each recursion, each polygon can be labelled with the value of its generator and these values mapped to create a piecewise continuous surface (choropleth map). By selecting vertices exclusively, an entire set of interpolated vales can be generated in one pass. Further, vertices may be considered as locally optimal sites for new data interpolation locations since they maximize the distance from triples of existing interpolation locations.

We implement these procedures in the Arc/Info 8.1 GIS platform and test them using empirical data consisting of a small number of spot heights sampled from a complex topographic surface. Our results suggest that the SRPVD procedure successfully enhances the resolution of sparse point samples thus enabling the generation of more accurate surface representations. In order to improve computational efficiency and data storage requirements while maintaining output accuracy, we also explore strategies for dealing with edge effects and for using subsets of $Q_{(k)}$ that satisfy tolerances specified for the spatial and/or the data value proximity of the generated points. The paper concludes with a discussion of planned extensions of the procedures described and potential application domains.

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