Surface Networks: Extension Of The Topology And Extraction From Bilinear Surface Patches

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Biography

Bernhard Schneider holds an MSc and a PhD (1998) in Geography . His research interests focus on digital elevation modelling, including modelling, interpolation, and uncertainty issues. He currently holds a position as Lecturer at the Departments of Geography and Earth Sciences at the University of Basel, Switzerland.

Surface Networks

Surface networks of topographic surfaces (Pfaltz 1976) are topological graphs of topographic objects. The graph's nodes are the passes, pits, and peaks of the surface $z = f(x, y)$, and the edges are represented by paths of steepest descent and ascent starting at the passes end ending at the pits and peaks, respectively. Paths of steepest descent are often called 'ravine lines' or 'valley lines', paths of steepest ascent are labelled 'ridge lines'.

For each pit, a drainage area can be identified bound by drainage area divides which are represented by sequences of ridge lines, passes, and peaks. Analogously, there exists a mountain for each peak, where each mountain is bound by a sequence of valley lines, passes, and pits.

From the elevations of the critical points, their x- and y-coordinates, and the lengths of the critical lines from passes to pits and peaks, weights can be calculated and assigned to the nodes and edges of the graph. Wolf (1990) calls this concept the Metric Surface Network and uses it for cartographic generalisation of topographic surfaces. The same information may also be used to estimate the morphologic significance of the topographic features. Furthermore, surface networks facilitate the delineation of watersheds and the detection of spurious pits.

Although the potential of surface networks has long been recognised (e.g., Pfaltz 1976), surprisingly few authors have actually presented algorithms for their extraction from digital elevation models (DEMs). Two notable publications on this topic, however, are Takahashi et al. (1995) and Wood (1998).

Takashiga et al. (1995) propose triangulating regularly distributed DEM points prior to the extraction of critical points and lines. The triangles are generated by subdividing each grid cell with one of its diagonals. Of the two possible configurations in each grid cell, Takashiga et al. choose the one with the smaller angles between the planes defined by the two resulting triangles. The triangle-based surface allows using simple algorithms for identifying critical points and for tracing paths of steepest descent and ascent. However, the subdivision of the cells into two planar triangles is based on a heuristic, and, thus, the extracted surface network is arbitrary to some degree.

Wood (1998) suggests specifying bi-variate quadratic surface patches at the DEM raster points. These surface patches allow the identification of principle axes and, thus, of the directions of steepest descent and ascent. This information is used to extract the critical points and to trace the critical lines. Furthermore, the bi-variate quadratic surfaces can be fitted to windows of any size. This enables performing the analysis on a desired level of scale. However, the approach does not guarantee topological consistency of the extracted network. Wood reports on passes being falsely connected to passes because intermediary pits were not identified. Also, the bi-variate quadratic surfaces tend to smooth the topographic surface with increasing window size. For instance, 'sharp' features (prominent breaklines) are not retained through the scales, and horizontal areas such as lakes shrink with increasing scale.

Extending The Conceptual Framework

Objects at the Edge of the Area of Interest

The delineation of drainage areas and mountains requests extraction of all valley and ridge lines forming the boundaries between the area objects. This, in turn, demands that all passes are found. Unfortunately, there exist, for instance, mountains that are separated by valley lines starting at passes that are located outside the area of interest. Since such passes are not identified, the valley lines separating the mountains are not found.

This example shows that consistency of the geometric representation of the Surface Network can not be guaranteed unless topographic objects are introduced that owe their existence to the edge of the area of interest. Such objects are termed "edge objects" and have not been considered in the literature so far. Edge objects can be critical points and critical lines (fig. 1). Edge pits are local minima of the surface at the edge of the area of interest. Edge peaks are defined analogously. Points at the edge of the area of interest that are

(i) local minima or maxima of the profile along the edge

(ii) but not local extreme points of the surface

are called edge passes. Local minima of the profile are called edge valley passes and mark the locations where a valley enters the area of interest. These points form the start of two ridge lines – both of which follow the edge of the area of interest – as well as the start of one valley line. The opposite valley line is located outside the area of interest and may be ignored.

Analogously, local maxima of the profile are called edge ridge passes and are related to crests rising into the area of interest. A ridge line as well as two valley lines start a this point.

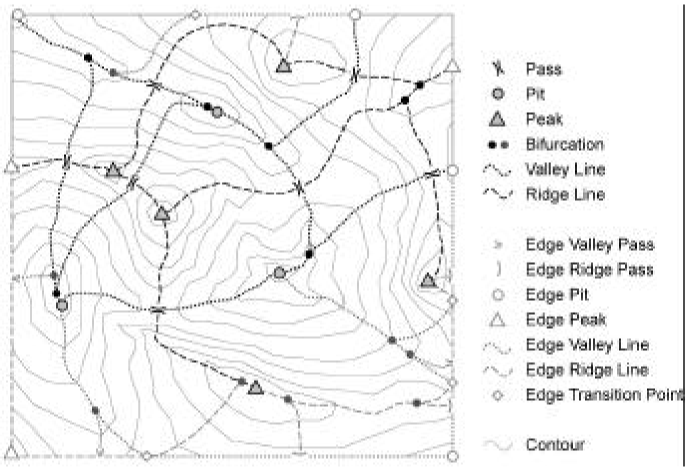

Fig. 1. Geometric representation of the Surface Network including edge objects.

Intersections of Valley and Ridge Lines

Wood and Rana (2000) report on the intersection of valley and ridge lines. They consider this intersection an artefact and suggest correcting it with the insertion of synthetic critical points. However, such intersections may indeed occur in reality under specific topographic settings. Explicitly considering these points as secondary critical features increases the information content of the model and facilitates the interpretation of the extracted graph.

At intersection points (fig. 2), the slope of the surface plotted as a function of the compass direction has two local minima (i.e., two locally maximal downwards slopes) and two local maxima (fig. 3). This means that both, the paths of steepest descent and ascent have not only an ordinary but also an alternate continuation at that point (fig. 2). Therefore, such points are termed bifurcations and may occur at the transition from a concave, e.g., a gulch, to a convex landform, e.g., an alluvial fan.

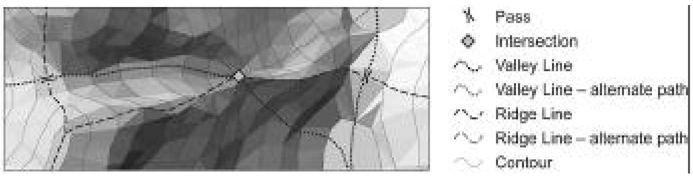

Fig. 2. Intersection between valley and ridge line. (To facilitate interpretation of the terrain, shaded triangles are drawn.)

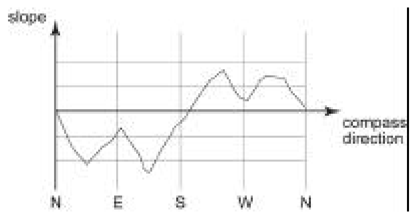

Fig. 3. Slope as a function of compass direction at the bifurcation point of figure 7.

Explicit Consideration of Horizontal Areas

Horizontal areas are frequent features of real terrain, and in digital representations of topographic surfaces, their number is even larger because, for instance, elevation values are rounded to a specific decimal place, or because the terrain model is generated by means of a TIN containing triangles with all corner points on the same contour.

Horizontal areas affect the extraction of surface networks in two ways. First, they may form critical points. Often, pits and peaks are represented in digital terrain models not as points but as horizontal polygons or horizontal lines. Likewise, horizontal polygons may have alternately higher and lower neighbouring regions so that they take the role of passes. Thus, it must be separated between the morphologic significance of topographic features and their geometric representation.

Second, a critical line may traverse a horizontal area (if, for instance, the area forms the horizontal section of a valley bottom) or start at a horizontal area pass. In these cases, a transit path and an exit point must be found. Both, however, depend on the (subjective) interpretation of the significance of the horizontal area. The horizontal area may be the digital representation of a lake, or it may be an artefact resulting from the model generation. The introduction of subjectivity and heuristics – seemingly a drawback of the explicit consideration of horizontal areas – ultimately offers the opportunity to optimize the suitability of the results for a subsequent application.

Extraction From Bilinearly Interpolated Raster Cells

The presented extraction method specifies the topographic surface from the DEM points with the help of bilinear surface patches. In each cell, the bilinear surface is uniquely defined by the four data points, and the global surface composed of bilinear surface patches is continuous (and continuously differentiable except along the cell borders). As a consequence, local minima and maxima of the global surface as well as passes are well-defined and can straightforwardly be detected. Pits and peaks only occur at the data points, while passes occur at data points as well as inside cells.

Critical lines either run along cell borders or through the cells. In the latter case, the bilinear surface may be interpreted as vector field consisting of gradient vectors. The path through a vector field is called streamline and can be calculated. In the case of bilinear surfaces, the streamline is a hyperbolic function.

If two adjacent grid points have the same elevation, they constitute a horizontal edge. Multiple horizontal edges may form horizontal edge groups that may include horizontal cells (where all four corner points have the same elevation). These horizontal edge groups constitute the horizontal areas described above. Whether an edge group is a critical point or a non-critical horizontal area can be evaluated by analysing the data points directly neighbouring the horizontal edge group. For instance, if all neighbouring data points are higher than the edge group, the horizontal area is a pit.

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