Sustainable Urban Land Use Allocation With Spatial Optimization

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Abstract

Sustainable urban form has been recognized as one of the major concerns of the planning practice. Current land use pattern trends with low-density, single-use, leapfrog urban growth on city outskirts call for a more efficient land use development strategies balancing economy, environmental protection, and social equity. In this paper, we present a new multiobjective spatial optimization model, which minimizes the conflicting objectives of open space development, infill and redevelopment, land use neighborhood compatibility, and cost distance to already urbanized areas. Land use allocation is restrained by a density based design constraint. Based on a hypothetical problem of 400 raster cells, we generate multiple exact compromise solutions with varying importance of allocation objectives. We discuss further model refinements and propose evaluating the prescribed patterns with multi agent geosimulation.

1. Introduction: Sustainable Land Use Patterns

Current urban land uses exhibit inefficient patterns that are of a major concern for sustainable development (Leccese et al, 2000; Silberstein and Maser, 2000; Ward et al, 2003; Williams et al, 2000). Low residential densities, sprawl and leapfrog fragmentation of urbanization, rapid open space development on the edge outweighing redevelopment of the declined inner city, and patches of single land use, all dominate in current urban form reality (Galster et al, 2001; Grimshaw, 2000; Silberstein and Maser, 2000; Williams, 2000). Such trends lead to an increasing ethnical and economic separation, deterioration of the environment, loss of agricultural land and wilderness, and the erosion of society's architectural heritage (Leccese et al, 2000, preamble). Research suggests that up to 70% of the consumed energy is dependent on land use arrangements (Barton, 1990). In consequence, the importance of sustainable land use allocation cannot be underestimated.

In this paper, we report findings of preliminary modeling experiments that utilize a new spatial optimization tool for sustainable land use planning. Motivated by a conflict-laden nature of urban activity allocation, we have designed a multiobjective sustainable land use allocation model that promotes infill development, balances conflicts of neighboring land uses, encourages accessibility to existing urban areas, and analyzes tradeoff between the conversion of undeveloped land and redevelopment. By testing the model on a hypothetical example, we generate a set of compromise spatial alternatives among the conflicting development goals. The patterns revealed in these options provide conceptual materializations of various aspects of the complexity of urban sustainability (Guy and Marvin, 2000).

Williams et al (2000) described a list of building blocks for sustainable urban form. These include urban layout and size, housing type, open space distribution, mix of uses, and various growth alternatives like intensification, extensification, or decentralization. They also characterize a sustainable urban form as the one that "enables the city to function within its natural and manmade carrying capacities, is user-friendly for its occupants, and promotes social equity" (Williams et al, 2000, p.4). Leccese et al (2000), in their *Charter of the New Urbanism*, described an analogous sustainable land use planning agenda. Their manifesto emphasizes infill development, mixed uses, compactness, and local geography as the main constituents of a balanced urban development. They highlight infill development as a means of conservation of environmental resources, economic investment, and social fabric. They stress neighborhood mixed-use compactness, which allows for locally embedded institutional and commercial activity. Finally, proper appreciation of local properties such as culture, ecological diversity, environmental factors, or building practice might also contribute to sustainable land use planning.

Many sustainable urban forms may stem from the above-mentioned postulates (Guy and Marvin, 2000). Moreover, these principles are spatially explicit in their majority, and therefore GIS-coupled spatial analysis and modeling may provide a potentially useful technology serving the planning practice of land use pattern sustainability. Consequently, we propose to define *sustainable urban land use allocation* as a normative model that recognizes and evaluates current land use pattern and introduces changes that promote compatibility of adjacent land uses, neighborhood compactness, infill development, and politically defensible redevelopment. Our definition focuses on land use intensification, which was recognized as the most promising form of urban intensification contributing positively towards sustainable cities, mainly because it reduces pressures of outward expansion (Williams, 2000). Compact neighborhoods with mixed uses proved useful for increased accessibility to city facilities for residents, and thus may contribute to promoting social equality (Masnavi, 2000).

Obviously, sustainability of land uses should be analyzed from various scale perspectives (Leccese et al, 2000; Silberstein and Maser, 2000; Ward et al, 2003). The *New Urbanism* movement divided the aspects of urban sustainability into scale-dependent areas of region, neighborhood/district, and street block (Leccese et al, 2000). The model we present here explicitly addresses spatial problems within the domain of neighborhood/district.

The remainder of this article is structured as follows. Section two provides a brief synopsis of existing spatial optimization models for land use allocation with the focus on their usefulness for our model. In section three, we formulate and describe the model. We develop a density based design constraint (DBDC) for compact neighborhood development, which promotes infill and counteracts a fuzzy urban-rural fringe. Section four reports the experimental problem we used in model evaluation. We report the verification of the assumptions underlying the normative model, through a tradeoff qualitative assessment among model objectives. The final section summarizes the research presented here and outlines future model refinements.

2. Optimization techniques for land use allocation

The utility of optimization as a normative tool for spatial problems is widely recognized (Arthur and Nalle, 1997; Church, 1999; Church, 2002; Chuvieco, 1993; Malczewski, 1999). These generative techniques allow for multiple scenario analysis, where the outcomes obtained are non-inferior or Pareto optimal to the objectives contained in the model (Cohon, 1978). Land use allocation problems comprise a subset of spatial optimization models, and involve efficient distribution of activities over feasible sites in order to meet demand and maintain physical, economic, environmental, or social constraints. Models involving allocation of spatial activities are not unique and span over such domains as urban and regional planning, forest management, reserve design, site restoration, facility location, land acquisition, or waste landfill siting (Aerts et al, 2003; Aerts and Heuvelink, 2002; Bammi and Bammi, 1979; Bammi et al, 1976; Benabdallah and Wright, 1992; Brookes, 2001; Brotchie et al, 1980; Chang et al, 1982; Cova and Church, 2000; Dökmeci et al, 1993; Gilbert et al, 1985; Minor and Jacobs, 1994; Nalle et al, 2002a, 2002b; Ward et al, 2003; Williams, 2002; Williams and ReVelle, 1996; Wright et al, 1983; Xiao et al, 2002). The majority of land use allocation models involve integer programming, where the variables are often binary, and represent two-choice decisions of whether or not to allocate a particular activity to a specific site (Malczewski, 1999).

The major shortcoming of most allocation models is the absence of existing land use patterns in model initialization (Church, 1999, 2002). The models usually convert completely undeveloped (green-field) areas, where every allocation of activity is new to the land under consideration (a revolutionary approach). This is a particularly flimsy assumption in urban planning, which by and large involves a modification of an existing situation (an evolutionary approach) and not building from scratch. "In brown-field planning (i.e. adding to, taking away, or transforming an existing configuration) there must be the capability to solve for a new configuration which maintains much of what currently exists and which adds or moves specific facilities to better locations" (Church, 1999, p.302). A valuable exception to green-field development is a reserve network design model by Nalle et al (2002), which extends an existing reserve pattern and also evaluates spatial efficiency of this scenario against an *open space conversion* case. The sustainable urban land use allocation model, which we present in this paper, builds upon the brown-field planning premise.

To the authors' knowledge, the only spatial optimization model that addresses urban sustainability is the regional scale model by Ward et al (2003). Their model allocates over time zoning options such as rural residential, urban residential, commercial, industrial, recreational, and special use to aggregate planning units based on regional population projections. Sustainability is addressed through incorporation of economic, social, or environmental requirements to the model, and minimization of deviations from these targets. The model produces fractions of residential use allocated to aggregate spatial units, and is further integrated with a local Cellular Automata (CA) model that assigns these zoning proportions to finer-grained spatial units based on several local scale suitability measures. While the Ward at al (2003) model presents a significant step towards modeling sustainable land use allocation, it is still inadequate in terms of addressing the variety of spatially explicit sustainability aspects (like contiguity, compactness, or infill development) mainly due to the fact that it is a regional model and these spatial characteristics may be obtained only indirectly through the integration with the CA model.

A variety of measures of sprawl development could be utilized in spatial optimization for sustainable urban activity allocation. As an example, Galster et al (2001) define the following characteristics (dimensions) of sprawl development: density, continuity, concentration, clustering, centrality, nuclearity, mixed uses, and proximity. Some of these indicators like compactness (a counterpart of Galster's 'clustering'), contiguity (a counterpart of Galster's 'continuity'), or accessibility (a counterpart of Galster's 'proximity'), are widely used in existing spatial optimization models. For example, contiguity is often described as the degree to which a specific use has been allocated to land in an unbroken fashion (Aerts et al, 2003; Galster et al, 2001; Williams, 2002; Wright et al, 1983). Likewise, compactness is defined as an allocation of like land uses next to or in direct proximity of each other, and may result in isolated roundish patches (Aerts et al, 2003).

We classified the existing contiguity and connectedness land use allocation constraints into three methodological categories: network based contiguity, edge based compactness, and adjacency based clustering. Network based contiguity constraints use various concepts of graph theory in quest of contiguous land patterns, where locations are nodes and their adjacency is represented with arcs. These include network flow (Shirabe, 2005), dual graph (Williams, 2002), and ordered closeness (Cova and Church, 2000). Edge based compactness optimizes the ratio or product of development perimeter length to a certain measure of total development area (Benabdallah and Wright, 1992; Gilbert et al, 1985; Minor and Jacobs, 1994). A variation of this approach is a core/buffer constraint that produces clusters of land uses surrounded by buffer zones (Aerts et al, 2003; Williams and ReVelle, 1996). Finally, clustering based on adjacency emerges from formulations that use concepts of direct topological touching of spatial units (Aerts et al, 2003; Fischer and Church, 2003; Wright et al, 1983).

In the sections that follow, we propose an alternative approach to encourage local clustering of land uses, based on the idea of core/buffer connectedness, and called a density based design constraint (DBDC).

3. A zero-one multiobjective model for sustainable land allocation

In light of the previous discussion, the major dilemma we encountered concerned the selection of proper objectives for model definition. Our approach at resolving the problem was to collect a comprehensive list of applicable objectives known from the literature and eliminating those that were inadequate for lack of their compatibility with our definition of sustainable urban development. One of such objectives was *maximization of single use clustering.* It was discarded due to the likely emergence of lumpy homogenous patches of land uses, which we actually want to avoid since they counteract mixed-use and globally compact development. Another objective – *density maximization* – does not necessarily positively influence urban sustainability, since it may cause overcrowding with its adverse social consequences like deterioration of individual quality-of-life. Furthermore, pure density maximization does not seem to have any direct meaning. Similarly, *mixed uses maximization* within one location does not seem reasonable. It would force the model to allocate as many uses to a given site as possible, sacrificing unnecessarily compatibility of nearby uses and increasing development and maintenance costs. We allow for use mixing over the area whenever it increases the objective value, but do not seek mixed uses maximization per se. Consequently, our model supports mixed uses over the neighborhood, but each individual location is labeled with one and only one land use.

Another common mathematical programming objective is *cost minimization,* which would involve land acquisition costs, initial development costs, and long-term operation costs (Dökmeci et al, 1993). To implicitly represent this objective, we assumed that it is always cheaper to allocate residential uses within new open land on urban fringe than redevelop (or infill-develop) within inner city. Another pragmatic reason for abandonment of this objective is the difficulty in obtaining accurate data about development and redevelopment costs for each individual location, and consequently high uncertainty of these datasets due to aggregation. *Contiguity* is binary in nature (a pattern is either contiguous or not) and thus its *maximization* does not make much sense physically (Wright et al, 1983). The traditional above-mentioned methods of addressing contiguity as a constraint were also discarded due to the widely recognized issue of computational tractability (Cova and Church, 2000; Shirabe, 2005; Williams, 2002). Similarly to the model of Williams and ReVelle (1996), the model presented below promotes spatial contiguity and compactness, but does not force them through these highly restrictive constraints.

Following this argument, we chose four objectives as suitable for model formulation:

1. Minimization of open space development that encourages efficient urban land utilization
2. Minimization of redevelopment that ensures economically defensible spatial change
3. Minimization of incompatibility of adjacent land uses that might prevent environmental deterioration, and
4. Minimization of distance to already developed areas, which acts as a coarse-equivalent to accessibility.

3.1 Model notation

The model was developed for a regular grid of cells. As already mentioned, the land use of each cell is homogenous. Also, we do not allow for urban land redevelopment leading back to open space, which is in practice very unlikely since once urbanized, land "typically stays that way" (Nalle et al, 2002b, p.60; Silberstein and Maser, 2000). Our model utilizes the concept of neighborhood, which we define as Moore neighborhood of range $r = 1$ (Weisstein, 2005).

Given these assumptions, consider the following notation:

j, = 1,2 ,…, n; where n is the total number of cells in the study area

l, m = $1,2,\ldots k$; types of urban land uses (single family residential, multi family residential, commercial etc.)

u = undeveloped land use type

D_l = set of cells that already have land use l

D = set of developed cells, all subsets of D are mutually disjoint

U = set of cells of undeveloped land $U \cup D = \bigcup$, where $\bigcup$ is the universal set of all cells under consideration

B_j = set of j's neighbors that are undeveloped

e_j = exiting land use of cell j

t_l = number of cells that initially have land use l, where $l = 1,2,\ldots k$

c_{lm} = estimated compatibility index between land use l and land use m (the higher this coefficient, the more compatible the land uses), $c_{lm} \in [0,1]$; if $l = m$, then $c_{lm} = 1$, in the model l is represented by d_j

d_j = dominant urban land use type within the neighborhood of j[1], The dominant land use type is the preferred (most compatible) land use to be allocated at j; $d_j = 1,2,\ldots k$ (urban land uses) or $d_j = u$, if the neighborhood is undeveloped[2]

s_j = number of initially developed cells within j's neighborhood

r_j = resistance to change for already developed location j, the higher the coefficient, the less probable that redevelopment occurs; $r_j \in [0,1]$

d_j = distance to the nearest developed area (in cells)

v_l = estimated demand for land use l (in number of cells)

b = minimum required number of neighboring cells that are developed after allocation

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[1] The dominant land use type within neighborhood is the one that covers the maximum neighborhood area. For regular grid, where area of each cell equals some constant value, we can determine d_j as the one having maximum number of neighboring cells (including self); for ties, the dominant land use is chosen randomly

[2] The dominant land use type is set to 'undeveloped' if and only if for a regular grid, all neighbors (including self) are undeveloped

Variables

$$x_{jum} = \begin{cases} 1, \text{ if undeveloped land at location } j \text{ is changed to } m; \\ 0, \text{ otherwise} \end{cases}$$

$$x_{je_jm} = \begin{cases} 1, \text{ if current land use } e_j \text{ at location } j \text{ is changed to } m; \text{ where } m \neq e_j \\ 0, \text{ otherwise} \end{cases}$$

$$x_{jlm} = \begin{cases} 1, \text{ if current land use } l \text{ land at location } j \text{ is changed to } m; \text{ where } m \neq l \\ 0, \text{ otherwise} \end{cases}$$

3.2 Model formulation

Minimize

$$\sum_{j \in U} \sum_{m} x_{jum} \tag{1}$$

$$\sum_{j \in D} \sum_{m \neq e_j} r_j x_{je_jm} \tag{2}$$

$$\sum_{j \in U} \sum_{m} (1 - c_{d_jm}) x_{jum} + \sum_{j \in D} \sum_{m \neq e_j} (1 - c_{d_jm}) x_{je_jm} \tag{3}$$

$$\sum_{j \in U} \sum_{m} d_j x_{jum} \tag{4}$$

Subject to

$$\sum_{m \neq e_j} x_{je_jm} \leq 1; \forall j \in D \tag{5}$$

$$\sum_{m} x_{jum} \leq 1; \forall j \in U \tag{6}$$

$$t_l - \sum_{j \in D_l} \sum_{m \neq l} x_{jlm} + \sum_{j \in (D - D_l)} x_{je_jl} + \sum_{j \in U} x_{jul} \geq v_l; \forall l \tag{7}$$

$$s_j + \sum_{i \in B_j} \sum_{m} x_{ium} \geq b \sum_{m} x_{jum}; \forall j \in U \tag{8}$$

$$x_{jlm} \in \{0,1\}; x_{jum} \in \{0,1\}; x_{je_jm} \in \{0,1\} \tag{9}$$

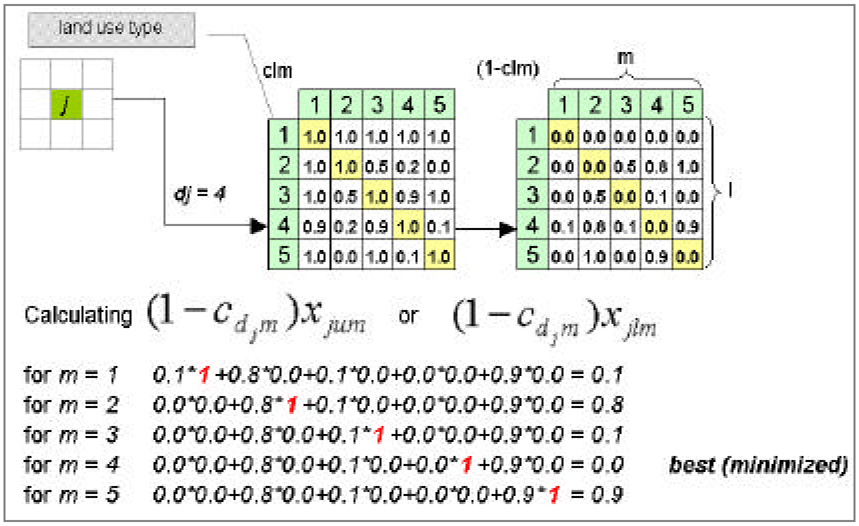
Number of decision variables	$kt_u + (k-1)\sum_{l} t_l$
Number of structural constraints (t_u – number of cells initially undeveloped)	$2t_u + \sum_{l} t_l + k$

3.3 Description of the model

With this change model, we seek to promote compactness, contiguity, and infill of urban development. The model supports redevelopment whenever it is politically defensible and economically reasonable. For each land use, its new location should be as close as possible to other, compatible land uses. Thus compactness and mixed uses may be achieved simultaneously. The coexistence of these elements of land use intensification is beneficial in terms of social equity (Burton, 2000).

The first two objectives allow for tradeoff evaluation between the minimization of the conversion of undeveloped land, and the minimization of redevelopment. Thus, assigning variable importance between these objectives, we can encourage either compact or diffuse growth (Ward et al, 2003).

Through minimizing redevelopment, we seek to minimize the change of current urban land use and therefore we encourage only reasonable redevelopment. For example, we could assign low resistance to change (r_j) values to derelict inner city sites, and hence increase the probability of redevelopment for these areas. Since undeveloped lands do not have the r_j value, their resistance to change implicitly equals 1 (which is the highest). Consequently, in our model any urban area with a lower r_j value ($r_j < 1$), has higher probability of land use change than the undeveloped land. Thus, giving open space areas the highest resistance to change, we 'penalize' allocation of urban uses to these areas and therefore encourage protection of undeveloped land. Objective (3), developed from Gilbert's et al (1985) concept of amenity and detractor cells, minimizes incompatibilities of development or redevelopment between site *j* and its neighborhood and thus addresses adjacent conflicts of land. The level of incompatibility is estimated between the candidate land use and the existing dominant land use d_j within *j*'s neighborhood (Figure 1). By assigning equal compatibilities for different land uses (e.g. both residential-residential and residential-commercial have compatibility of 1), this objective promotes mixing of adjacent uses (Table 3). The last objective (4) minimizes the distance of new development to already developed sites.



	1	2	3	4	5
1	1.0	1.0	1.0	1.0	1.0
2	1.0	1.0	0.5	0.2	0.0
3	1.0	0.5	1.0	0.9	1.0
4	0.9	0.2	0.9	1.0	0.1
5	1.0	0.0	1.0	0.1	1.0

	1	2	3	4	5
1	0.0	0.0	0.0	0.0	0.0
2	0.0	0.0	0.5	0.8	1.0
3	0.0	0.5	0.0	0.1	0.0
4	0.1	0.8	0.1	0.0	0.9
5	0.0	1.0	0.0	0.9	0.0

Calculating $(1-c_{d_j m})x_{jum}$ or $(1-c_{d_j m})x_{jlm}$

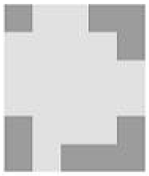
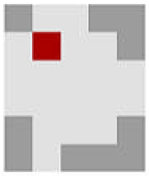
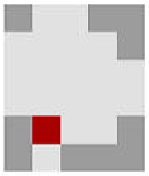
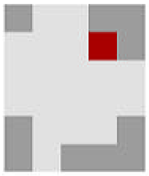
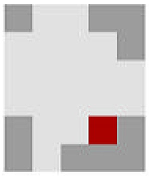
for $m = 1$ 0.1*1 +0.8*0.0+0.1*0.0+0.0*0.0+0.9*0.0 = 0.1
for $m = 2$ 0.0*0.0+0.8*1 +0.1*0.0+0.0*0.0+0.9*0.0 = 0.8
for $m = 3$ 0.0*0.0+0.8*0.0+0.1*1 +0.0*0.0+0.9*0.0 = 0.1
for $m = 4$ 0.0*0.0+0.8*0.0+0.1*0.0+0.0*1 +0.9*0.0 = 0.0 **best (minimized)**
for $m = 5$ 0.0*0.0+0.8*0.0+0.1*0.0+0.0*0.0+0.9*1 = 0.9

Figure 1 Minimizing incompatibility between current dominant land use and the potential land use to be allocated. For dominant land use type $d_j = 4$, we calculate its incompatibility ($1 - c_{djm}$) for all potential new land uses *m* to be allocated to *j*. Then, for every *m* we calculate its objective value (**1** means we picked this land use for allocation).

Constraints (5) and (6) ensure that we can allocate maximally one land use to each cell *j*. Equation (7) guarantees that the demand for land use *l* is satisfied. This constraint not only permits allocation of land use to undeveloped land but also to already urbanized areas (Lemberg and Church, 2000).

Equation (8) represents the density based design constraint (DBDC). It ensures that we will allocate to a given cell *j* if and only if the sum of the cell's initially and newly developed neighbors is at least equal to a threshold value *b* (Table 1). Therefore, the higher the value of *b* in this constraint, the more compact and contiguous is the pattern obtained and thus leapfrog development is prevented. DBDC, combined with the compatibility objective, is a surrogate to traditional zoning in urban planning.

Table 1 Density based design constraint (equation 8). With all other objectives and constraints unchanged, the new land use (dark color) is allocated to the neighborhood that meets the value of *b*

Input pattern	b=1	b=2	b=3	b=4
				

The maximum value of *b* depends on the size of the neighborhood considered. For example, if the neighborhood is a depth of one cell about a focus cell *j* (Moore neighborhood with $r = 1$; Weisstein, 2005), there are eight neighbors. Thus, an absolute maximum size of *b* cannot be larger than 9, if you include the focus cell as well. There will always be a set of cells on the perimeter of a given allocated land use (boundary cells). Observe the situation when the focus cell is on the perimeter of a given cluster of cells allocated to a given land use. The worst case or the case in which the focus cell has the fewest developed neighbors is when the cell is in the corner of the perimeter. If it is the corner cell then at most three other cells of urban land use can be allocated within its neighborhood. Thus, counting the focus cell as well, the value of *b* could be no larger than 4, which is the worst case for high connectivity. Hence, if $r = 1$, then the neighborhood will be a $2r+1$ by $2r+1$, which is 3x3, in size. The maximum possible size of *b* for a corner cell is then $r+1$ by $r+1$. Thus, if we had a neighborhood size of $r = 2$ or two cells deep about the focus cell *j*, then the neighborhood would be a 5x5 (i.e. $2r+1 = 5$) and the maximum size of *b* is 9, which represents 3x3 ($r+1 = 3$). The model could be executed with larger *b* values, but there is no guarantee that a feasible solution exists, since in such cases the constraint forces inward development and forbids development within the neighborhood of the boundary cells. This condition is especially true when the problem assumes no existing developed land.

Finally, equations (9) guarantee that the decision variables are binary.

4. Model evaluation

Model assessment took into consideration the following questions:
Under what conditions the obtained pattern is compact and contiguous?
What values of *‘b’* in DBDC intensify the level of infill development?
In what circumstances the compatibility of allocated land uses to adjacent cells is maintained?
What is the degree of redevelopment?

4.1 Initialization and solution

The model was tested against a hypothetical example that covered an area of 20x20 cells (Figure 2). The set of land uses under consideration comprises commercial, industrial, residential, recreational, and undeveloped. The last two types are a special case. The recreational land use acts as a substitute for *preserved* areas excluded from redevelopment. The *undeveloped* type represents open space areas that might be considered for build-out. The accompanying input datasets are presented in figure 3. The ‘Number of developed neighbors’ and ‘dominant neighborhood land use’ layers were derived from input land use theme using Python scripting. The ‘Number of developed neighbors’ was obtained by counting developed cells within the neighborhood of cell *j*. The ‘Dominant neighborhood land use’ was established based on the ‘dominant rule’ described in section 3.1. The ‘Resistance to change’ for developed cells was generated randomly. Finally, distance to developed cells was obtained with ArcGIS 9.0 Spatial Analyst Euclidean Distance function (ESRI, http://www.esri.com/). The distance is given in ‘cell’ units.

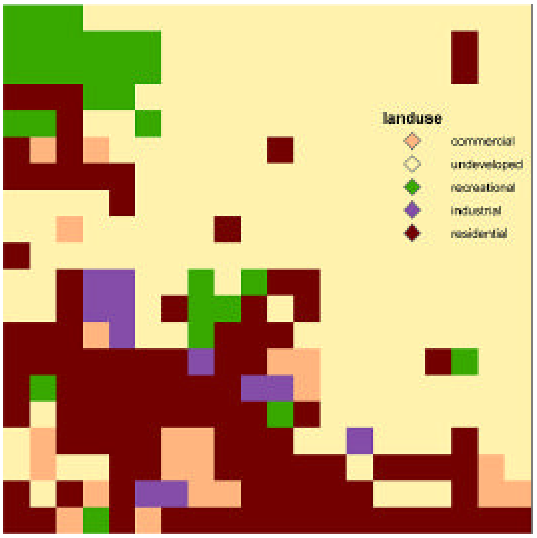


Figure 2 Initial land use pattern of the experimental 20 by 20 grid

The demand for allocation (Table 2) and land use compatibility (Table 3) were established arbitrarily. We used the weighting method (Cohon, 1978) to vary the importance of objectives and analyze tradeoffs. For the DBDC, the *b* value was set to 0,1,2,3,or 4. Problems were formulated using the MPS file format (standard format for Linear and Integer Programming), and generated with Python scripting. The problems contained 978 decision variables.

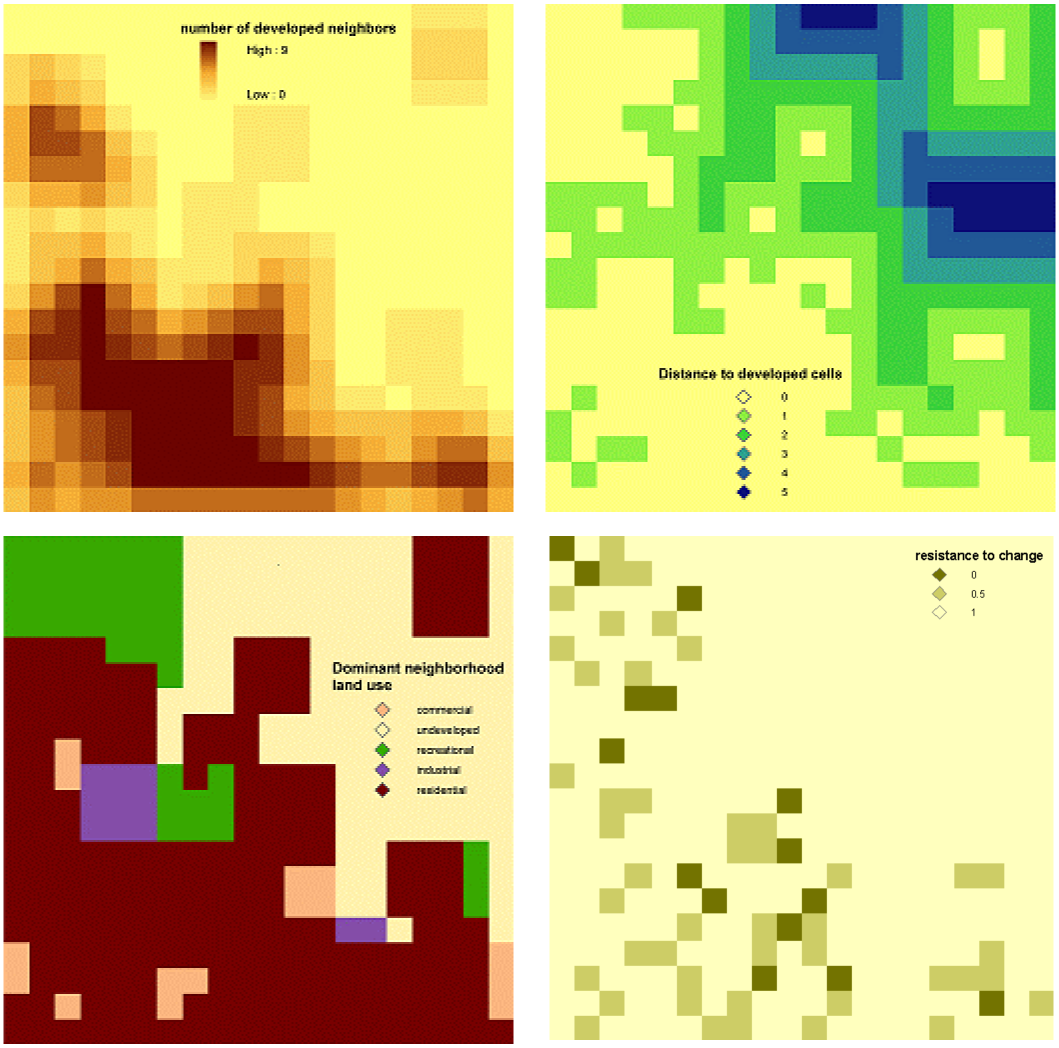


Figure 3 Input datasets

Table 2 Demand for allocation

Land use	Current number of cells	Demand
Commercial	21	31
Industrial	11	16
Residential	103	163

Table 3 Compatibility between adjacent land uses

Land use	Commercial	Recreational	Industrial	Residential
Undeveloped	1.0	1.0	1.0	1.0
Commercial	1.0	0.8	0.8	1.0
Recreational	0.8	1.0	0.0	1.0
Industrial	0.8	0.0	1.0	0.0
Residential	1.0	1.0	0.0	1.0

Problems were solved with exact branch-and-bound method of Linear Integer solver in Lingo 9.0 extended version, produced by LINDO Systems, Inc. (http://www.lindo.com/, 2004), on a Mobile Intel Pentium(R) CPU 3.06GHz and 448 MB RAM. The solutions obtained were globally optimal, and the maximum solution time was 85s. We observed an increased solution time and number of solver iterations for high values of ‘b’ in DBDC (Table 4).

Table 4 Solver iterations with increasing value of constraint (8)

Value of ‘b’	Total Solver Iterations
0	1
1	1
2	27
3	206
4	36846

4.2 Results and discussion

Figure 4 shows a matrix of the results for different optimization runs. Each row contains patterns obtained for a fixed set of preference weights for objectives and an increasing value of ‘b’ in DBDC (e.g. 0001 means that all objectives except the 4th are unimportant).

We followed the approach by Cohon (1978), and started our analysis from optimizing each objective individually (Row 1 to Row 4). After this extreme case, we applied a systematic variation of nonnegative weights for objectives to find combinations of preferences that would address the questions about pattern compactness and contiguity, level of infill and redevelopment, and adjacent use compatibility (Figure 4 Row 5, Table 5).

A rough exploration of the acquired patterns reveals that low values of *‘b’* in BDBC may result in a more erratic pattern regardless of the preferences of objectives. More importantly, such values produce more undeveloped wastelands in the city. Thus, high level of infill occurs for high *‘b’*. As an example, consider patterns in Table 5. For *b* = 4, there exists only one undeveloped cell within the largest patch of development, whereas *b* = 2 results in 7 such cells.

Leapfrog development is also restrained with a more constraining DBDC. We can easily observe the small ‘island’ of development in the NE corner which shrinks with the increase of *‘b’* value (Figure 4, Table 5). Obviously, another important factor in producing more compact clusters is the distance to development (Figure 4, Row 1).

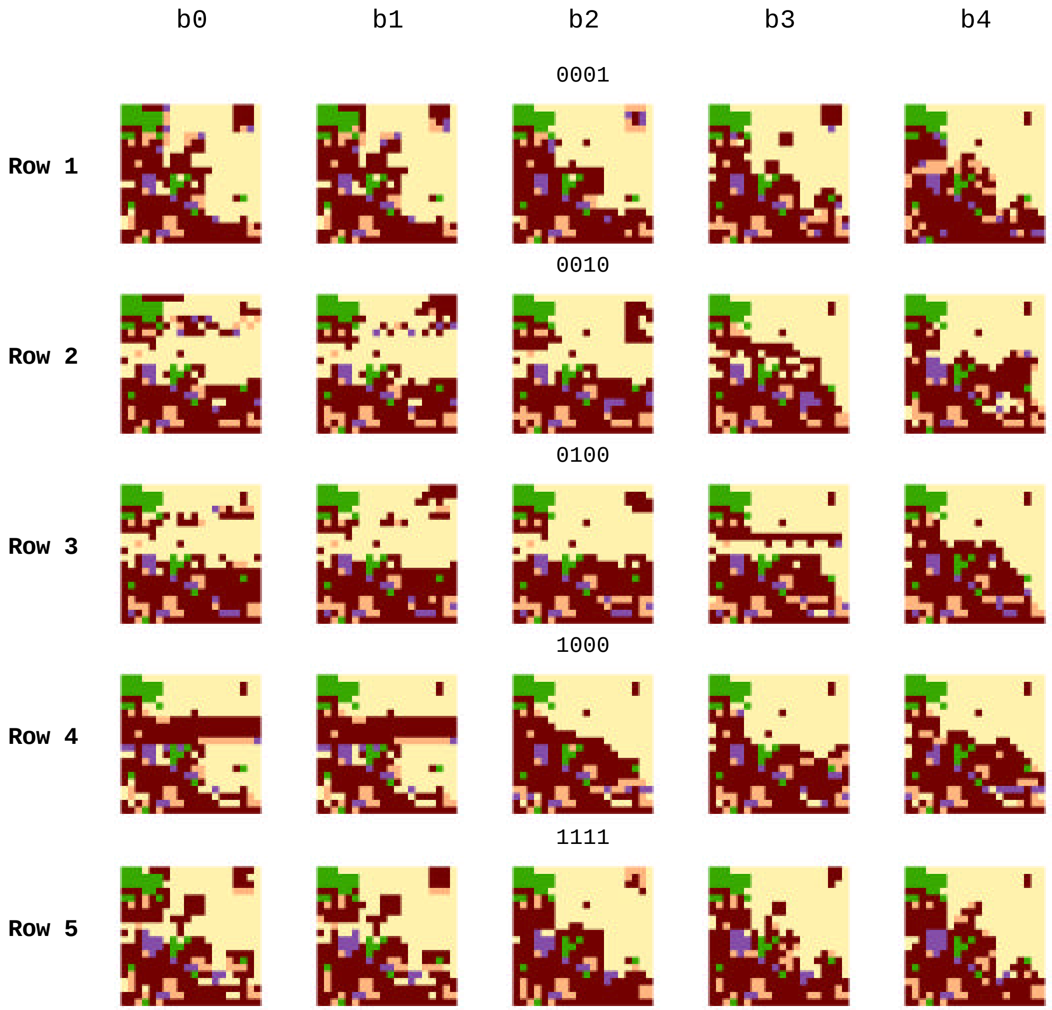


Figure 4 Comparison of land use patterns with the increasing value of constraint 8 (from the left column to the right), row headers represent the weighting combinations assigned to objectives

The only real detractor to other land uses was the industrial type (Table 3). Hence the examination of incompatibility focused on such undesirable combinations of allocations as *industrial – residential* or *industrial – recreational*. Table 5 contains screenshots of patterns with high importance of the 'minimizing incompatibility' objective. The adjacencies for allocated uses seem compatible (remember that we do not force changes for already developed areas and thus acknowledge such existent incompatibilities). We also observed that the DBDC constraint has an influence on compatibility i.e. its low values might encourage incompatible adjacencies.

Table 5 High importance of compatibility between adjacent land uses

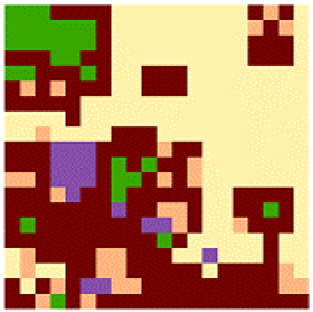
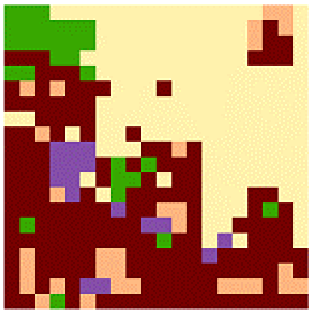
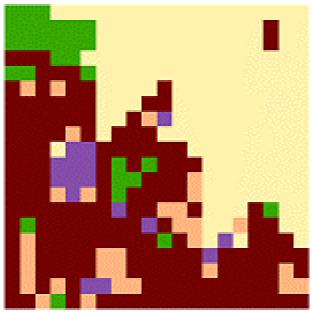
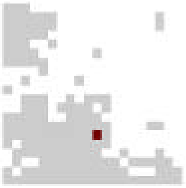
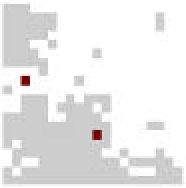
b = 0	b = 2	b = 4
		

Table 6 With increasing importance of *minimizing open space* development more sites are redeveloped (dark color)

Minimizing redevelopment is more important than minimizing new development	Minimizing new development is more important than minimizing redevelopment
	

The final aspect is the degree of redevelopment. We recognize that obtaining sustainable land use patterns is in practice highly constrained by the economical and political viability of such decisions. We believe that the minimization of the impact of change should be considered in any normative model for planning; otherwise these models will not gain any widespread use (Lemberg and Church, 2000). The redevelopment level we obtained in our hypothetical example (Table 6) might be considered satisfactory, given the size of the problem (400 cells).

In general, the preliminary tests of our model produced very plausible outcomes. The model introduces changes that prevent leapfrog skittish land use patterns, reclaims abandoned areas in the 'inner city', meets the compatibility requirements, and imposes reasonable redevelopments for urban areas.

5. Future work

The model reported in this paper is a part of a larger research activity. This section describes the research context and outlines future model refinements.

5.1 Model refinements

We hypothesized that the more checkerboard the pattern of developed areas, the more sustainable the city in terms of mixed use, but the less sustainable in terms of quality-of-life (due to increased incompatibilities). The size of the problem presented here neither confirmed nor rejected this hypothesis, regardless of the large number of optimization runs (about 150). Therefore, the next step will involve testing the model on a much larger real-world application, and investigating the limits of exact and heuristic solution techniques (Aerts and Heuvelink, 2002; Benabdallah and Wright, 1992; Brookes, 2001; Hillier and Lieberman, 2005; Nalle et al, 2002b; Williams and ReVelle, 1996; Xiao et al, 2002).

The *cost distance* objective – equation (4) – should be further refined to address variable accessibility needs for different activities. Rather than using the absolute distance to development regardless of land use type, we will introduce distance minimization of new development to associated needs like residential to industry or commercial, or residential to nearby residential.

Current DBDC only considers whether a neighbor is developed or not. It neglects the neighbor's use compatibility. In future versions, the constraint can be extended to ascertain some level of neighborhood compatibility. For example, for $b = 3$ (minimum 3 neighbors developed) at least 2 neighbors must be compatible. Such constraint would force some predefined level of compatibility for the allocated land use, instead of just optimizing the incompatibility objective, which does not guarantee that the adjacent uses are compatible (Arthur and Nalle, 1997).

We recognize that our model is not dynamic and therefore does not account for time interdependencies. Time dependent optimization models are scarce and dominate in timber harvest scheduling problems (Church and Barber, 1992; Shirabe, 2004). Yet, the model presented in this paper is specifically designed as a prescriptive and not descriptive tool, and hence produces design blueprints for further evaluation in spatial decision-making. We focused on generating sustainable patterns and not on the underlying processes that might result in such patterns. The latter will be addressed using a dynamic technique of Multi Agent Geosimulation (Benenson and Torrens, 2004).

5.2 Larger research context

The sustainable land use allocation model is a part of a larger project of investigating spatial behaviors of developers and residents that could lead to sustainable city patterns. The results of the optimization model may be used as a comparative benchmark for assessment of disparate spatial residential patterns obtained from various behavioral configurations used in an agent-based model. The research would address the question of whether such optimized sustainable patterns are possible to achieve in practice through land use policies.

6. Conclusions

The complexity of sustainability in urban land use patterns has been widely debated in city planning. In this paper, we presented a mathematical programming model that addresses the problem with optimization of such sustainability objectives like new development, redevelopment, land use compatibility, and accessibility. Additionally, the model may encourage infill development with the density based design constraint. It might be argued that the reported method of achieving sustainable cities in too analytical and rigorous. Although such approach may be perceived as too restrictive for ill-defined urbanization problems, we believe that spatial optimization has a great potential in managing urban form of haphazardly growing cities. With properly defined models, we are guaranteed that the solutions we deliberate on are the best that can be achieved within the model context. Such patterns present some idealized frames of reference, which allow for exploratory analysis of the current – highly inefficient and dissatisfactory – land use arrangement situation, and reveal possible improvements of the urban environment we live in.

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