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Incorporating both Geographical Space and Attribute Space into the Kernel Weighting Function of Geographically Weighted Regression

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1 **Abstract**

2 Geographically Weighted Regression (GWR), following the general principle of local smoothing
3 and locally weighted regression, has been developed to study the spatial heterogeneity in a
4 regression context. The kernel weighting function is the key component used to account for the
5 spatial heterogeneity in GWR. The spatial heterogeneity generally results from both
6 “geographical space” and “attribute space”. However the current kernel weighting function only
7 considers the geographical distances of the neighbors from a focal point in the study area, while
8 the attributes of the focal point and its neighbors are totally ignored. In this study, we proposed a
9 new kernel weighting function that combines the “geographical space” and “attribute space”
10 between the focal point and its neighbors such that (1) neighbors with greater geographical
11 distances from the focal point are assigned smaller weights, and (2) at a given geographical
12 distance, neighbors with similar sizes to the focal point are assigned larger weights. The
13 characteristics of the new weighting function are investigated with four tree attributes, diameter
14 at breast height (DBH), DBH^2 , area potentially available (APA) and Hegyi’s competition index
15 (CI), and the new weighting function is also tested using three simulated forest stands with
16 different spatial patterns. The results indicate that smaller model residuals and better predictions
17 can be obtained from the GWR model with the new spatial-attribute kernel weighting function
18 than that from the traditional GWR model.

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20 **Key words:** Geographically weighted regression, spatial pattern analysis, spatial heterogeneity,
21 locally weighted regression.

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Introduction

Local modeling has become an increased interest in recent years. A number of approaches, such as spline functions and kernel regression, have been developed for examining local relationships in nonspatial data (Wahba 1990; Green and Silverman 1994; Cleveland and Devlin 1988). In spatial data, geographically weighted regression (GWR) has become popular for depicting the spatial heterogeneity in a regression context in recent years (Brundson et al. 1996; Fotheringham et al. 2002; Zhang and Shi 2004). In GWR, any spatial heterogeneity in the relationship is accounted for by the local estimation of model coefficients through a spatial weighting function. This spatial weighting function is a decreasing function of distance (geographical space) from the focal observation (x_0) so that the impact of the neighbors (x_i , $i=1\dots k$, k is the number of neighbors) nearby is stronger than those farther away. In general, spatial data consist of both attribute and spatial information (e.g., spatial coordinates; Fotheringham et al. 2002). GWR uses only the distance (geographical space) to determine the weights. It may not be realistic and reasonable, because the effects of the attributes of the focal observation and its neighbors are totally ignored. In other words, no matter how large or small of the attributes of its neighbors, if they have the same geographical distance from the focal observation, they would have the same weight. Apparently, the attribute information should be considered in the weighting function.

The development of GWR follows the general principle of local smoothing and locally weighted regression (Leung et al 2000; Páez et al 2002), in which the weights are determined by the size of the residuals (Cleveland 1979; Cleveland and Devlin 1988; Casetti 1982; Casetti and Can 1999). For a given focal point x_0 in the locally weighted regression, if the sizes of its neighbors x_i are similar to the size of x_0 , the “distance of attribute space” between x_0 and x_i is

1 small. These neighbors are assigned large weights by the weighting function. In contrast, its
2 neighbors x_i , with the sizes dissimilar to the size of the x_0 , are assigned small weights because
3 they are far away from x_0 in the “distance of attribute space”. In other words, the weights are
4 determined by the “attribute space” instead of the “geographical space” (Leung et al. 2000). This
5 approach pays more attention to the fitting of the dependent variable rather than on spatially
6 varying parameters. However, the “attribute space” approach does not consider geographical
7 locations of the neighbors and the relative distance (geographical space) between x_0 and x_i .

8 In this study, we propose a new approach that will incorporate the attribute of the
9 observations into the spatial weighting function used in GWR. The new weighting function will
10 combine the “geographical space” and “attribute space” between the focal point (x_0) and the
11 neighbors (x_i) such that (1) the neighbors (x_i) with large geographical distances from x_0 will be
12 assigned small weights, and *vice versa*, and (2) at a given geographical distance, the neighbors
13 (x_i) with similar attributes to x_0 will be assigned large weights, and *vice versa*. The properties of
14 the “spatial-attribute” weighting function were tested with simulated forest stands (see Data
15 section) with regard to spatial continuity as well as statistical and biological interpretation.

16 One attribute and three attribute functions were used once at a time in the spatial-attribute
17 weighting function. The attribute used in this study is the tree diameter at breast height (DBH).
18 The three attribute functions are DBH^2 and two traditional competition indices (CI), Hegyi’s CI
19 (Hegyi 1974) and area potentially available (APA; Brown 1965; Moore et al 1973). In general,
20 the tree attribute is defined as the measurable tree characteristics (e.g., DBH). However, for
21 simplicity, we defined these three attribute functions as tree attributes in this study. DBH^2 is
22 proportional to the tree basal area. The selection of these attributes is to test whether different
23 tree characteristics can alter the model performance. The reason of choosing APA and Hegyi’s

1 CI is that they have been widely used in forest growth and yield models (Newton and Jolliffe
2 1998; Shi and Zhang 2003), and demonstrated that they are useful indices for measuring tree
3 competition (Moore et al. 1973; Biging and Dobbertin 1995). Another reason is that a large
4 Hegyi's CI means the subject tree has relatively strong competition from its neighbors, however
5 a large APA indicates that the subject tree has stronger competition than its neighbors.
6 Therefore, these two CIs have opposite meaning for interpreting tree competition. It is useful to
7 test the performance of the spatial-attribute weighting function.

8 The objectives of this study were (1) to generate three example plots with different spatial
9 patterns (i.e. regular, random, and clustered) of tree locations, (2) to model the relationship
10 between tree size and growth using the GWR methodology with different weighting functions
11 (i.e. spatial weighting and spatial-attribute weighting functions), and (3) to compare and evaluate
12 the performance of the two weighting functions for modeling the effects of spatial heterogeneity
13 on tree growth.

14 **Data**

15 Three example plots used in this study were generated using a process-based stand model
16 AMORPHYS (Valentine et al. 2000). This public-domain software is developed by USDA
17 Forest Service Northeastern Research Station and can be downloaded from the web site
18 "http://ftp.fs.fed.us/ne/durham/4104/products/InstallAMORPHYS.ex_". AMORPHYS can (1)
19 generate the locations of model trees, (2) sample tree diameters from a target distribution (e.g., a
20 two-parameter Weibull) and assign those diameters to model-tree locations, (3) compute other
21 tree attributes such as height and crown length, and (4) predict tree growth (Valentine et al.
22 2000). The generation of tree locations (coordinates) is based on an algorithm called LPOINT
23 (Penridge 1986) that can produce two-dimensional point patterns from regular, through Poisson

1 or random, to strongly clumped, depending on the values of two parameters, λ and μ . λ is the
 2 randomness parameter and μ is the mean point density (for detailed information, see Penridge
 3 1986).

4 We generated an example plot for each of three spatial patterns, i.e. regularity,
 5 randomness, and clustering. Each example plot was a 100 x 100 m square plot. The initialization
 6 of the plots was manually set up for the model (Table 1). The specifications of the model
 7 parameters were (1) regular plot: $\lambda = 0.45$ and $\mu = 1.0$, (2) random plot: $\lambda = 1.0$ and $\mu = 1.0$, and
 8 (3) cluster plot: $\lambda = 10.0$ and $\mu = 5.0$. Figure 1 shows the map of tree locations for the three
 9 example plots. Then tree initial diameters were obtained from a Weibull distribution and were
 10 assigned to the tree locations. The trees in the three plots were projected for a growth period of 5
 11 years. The descriptive statistics of tree initial diameter at breast height (DBH) and 5-year basal
 12 area growth (BAG) were listed in Table 2.

13 Methods

14 (1) GWR Model

15 Suppose we have a set of n observations $\{X_{ij}\}$ with the spatial coordinates $\{(u_i, v_i)\}$, $i = 1,$
 16 $2, \dots, n$, on p independent or predictor variables, $j = 1, 2, \dots, p$, and a set of n observations on a
 17 dependent or response variable $\{y_i\}$. The underlying model for GWR is

$$18 \quad y_i = \mathbf{b}_0(u_i, v_i) + \sum_{j=1}^p X_{ij} \mathbf{b}_j(u_i, v_i) + \mathbf{e}_i \quad [2]$$

19 where $\{\beta_0(u_i, v_i), \beta_1(u_i, v_i), \dots, \beta_p(u_i, v_i)\}$ are $p+1$ continuous functions of the location (u_i, v_i) in
 20 the study area. The e_i is the random error term with a distribution $N(0, s^2 I)$. Suppose $W_i(u_i, v_i)$ is

$$21 \quad W_i(u_i, v_i) = \begin{pmatrix} w_{i1} & 0 & \cdots & 0 \\ 0 & w_{i2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & w_{in} \end{pmatrix} \quad [3]$$

1 The estimator of β_i is given by GWR:

$$2 \quad \hat{\mathbf{b}}_i = \left(X^T W_i(u_i, v_i) X \right)^{-1} X^T W_i(u_i, v_i) y \quad [4]$$

3 After the GWR model regression, a set of parameter estimates can be obtained for each data
4 point. The weights (w_{ij}) in the weight matrix $W_i(u_i, v_i)$ is a decreasing function of distance d_{ij}
5 between subject i and its neighboring location j . In general, the spatial weighting function is
6 taken as the exponential distance-decay form:

$$7 \quad w_{ij} = \exp\left(-\frac{d_{ij}^2}{h^2}\right) \quad [5]$$

8 where h is called kernel bandwidth. If locations i and j coincide (i.e., $d_{ij} = 0$), w_{ij} equals one;
9 while w_{ij} decreases according to a Gaussian curve as the distance d_{ij} increases. However, the
10 weights are nonzero for all data points, no matter how far they are from the center i

11 (Fotheringham et al. 2002). The kernel bandwidth can be determined by (1) predefined kernel
12 bandwidth; (2) Minimum Akaike Information Criterion (AIC); (3) cross-validation procedure;
13 (Fotheringham et al 2002; Zhang and Shi 2004).

14 **(2) Spatial-Attribute Weighting Function**

15 The spatial weighting function (Equation [5]) only takes the geographical distance into
16 account, and ignores the influence of the observation's attributes. Since both tree size and
17 location have strong impacts on competition among trees, crown structure, growth, and mortality
18 (e.g., Miller and Weiner 1989; Moeur 1993; Newton and Jolliffe 1998), we propose to modify
19 Equation [5] as follows:

$$20 \quad w_{ij} = \exp\left(-\frac{d_{ij}^2}{h^2} \times f(\mathbf{t})\right) \quad [6]$$

1 where $f(\mathbf{t})$ is a function that changes the weight w_{ij} according to the difference (\mathbf{t}) between the
 2 size of the focal tree and its neighbors.

3 Following the idea of the weighting function in locally weighted regression techniques
 4 (Cleveland 1979; Castti 1982; Cleveland and Devlin 1988), the weight should decrease as the
 5 difference between the focal observation (i) and its neighbors (j) increases. The symmetric
 6 weight is one of the important properties of the weighting function because it reduces bias
 7 (Cleveland and Devlin 1988). The $f(\mathbf{t})$ function can be bisquare, “tribcube”, or exponential
 8 functions, therefore we propose the following format for the $f(\mathbf{t})$ function:

$$9 \quad f(\mathbf{t}) = \exp \left(\left| 1 - \frac{ATT_{ij}}{ATT_{ii}} \right| \right) \quad [7]$$

10 where ATT_{ii} is the attribute of the focal tree i , and ATT_{ij} is the same attribute of the neighboring
 11 tree j . In this study, the tree attributes include DBH, DBH^2 , Hegyi’s CI and APA. APA requires
 12 construction of Voronoi polygons (via the Dirichlet tessellation) around each tree. The distance
 13 between each tree is bisected at right angles, and these successive right angle lines are joined to
 14 form a polygon. Hegyi’s CI is defined as:

$$15 \quad \text{Hegyi's CI} = \sum_j^n (c(h)DBH_j / DBH_i) / L_{ij} \quad [8]$$

16 where i denotes the subject tree, j denotes the neighboring competitor, DBH is the diameter at
 17 breast height, $c_{ij}(d)$ is a spatial matrix for a given bandwidth (h), n is the number of neighbors
 18 inside a neighborhood zone of the subject tree, L_{ij} is the distance between tree i and tree j .

19 The spatial-attribute weighting function (Equation [6]) takes into account both the
 20 geographical distance and the attribute of the focal tree and its neighbors. If the tree attributes
 21 (e.g., DBH) of the neighboring trees are greatly different from a given focal tree, the smaller
 22 weights are assigned to these neighboring trees. In contrast, if the differences of tree attributes

1 between the focal tree and its neighbors are small, larger weights are assigned according to
2 Equation [6]. In the case that the size of a neighboring tree is the same as the focal tree (i.e.,
3 $f(\mathbf{t})=1$), the weight for that tree is determined by the spatial distance only. Biologically, it implies
4 that the competition is a reciprocal process. Large trees have influence on small trees, however
5 small trees also compete for resources with large trees.

6 **(3) Regression Model**

7 Many models for tree diameter or basal area growth have been developed over the past
8 several decades. Different models work well under different conditions (Vanclay 1994). In this
9 study, we chose the following linear regression model to investigate the differences between the
10 spatial weighting function (Equation [5]) and the spatial-attribute weighting function (Equation
11 [6]) used in the GWR model.

$$12 \quad \log(BAG + 1) = \mathbf{b}_0(u, v) + \mathbf{b}_1(u, v) \cdot \log(DBH) + \mathbf{b}_2(u, v) \cdot DBH^2 + \mathbf{e} \quad [8]$$

13 where BAG is the basal area growth, DBH is the initial tree DBH, \log is a 10-based logarithm,
14 $\beta_0(u, v) \sim \beta_2(u, v)$ are regression coefficients to be estimated, and e is the model random error. If
15 the spatial coordinates are removed from the above model, Equation [8] becomes the derivative
16 model of the Bertalanffy growth function, which has been used as a basic function in several
17 forest growth and yield models due to its simplicity and robust predictions (e.g., Wykoff 1990;
18 Hann and Larsen 1991; Vanclay 1994; Monserud and Sterba 1996).

19 **(4) Model Comparison and Evaluation**

20 The GWR models with the spatial and spatial-attribute weighting functions were
21 compared based on the criteria of the root mean square error (RMSE) and bias (Maltamo et al.
22 1995) such as

$$RSME = \sqrt{\frac{\sum_i (BAG_i - BA \hat{G}_i)^2}{n}} \quad [9]$$

where n is the number of observations, BAG_i is the basal area growth (BAG) of the i th tree, $BA \hat{G}_i$ is the predicted BAG of the i th tree.

Model residual (RS = observed - predicted) and absolute model residual (ARE = |observed - predicted|) were also evaluated across diameter classes. Then paired t-tests were used to compare the differences of RS and ARE between the GWR models with the two weighting functions. Finally, the contour plots of the parameter estimates from the two weighting functions were mapped for spatial assessment.

Results

(1) Determination of Kernel Bandwidth

In this study, we decided to predefine the bandwidth for the three example plots based on the variogram of the OLS model residuals. Because of the relatively small size of the example plot, reasonable kernel bandwidth can not be obtained using the cross-validation procedure and minimum AIC approach. The variogram curves indicate that the ranges of these variograms (Figure 2) for the three example plots were 10m (regular plot), 7m (random plot), and 6m (clustered plot), respectively. Since the range of a variogram indicates that there was no spatial autocorrelation between trees beyond the distance (Isaaks and Srivastava 1989; Kohl and Gertner 1997), we chose these three range distances as the bandwidth for the three example plots. In addition, similar distances (or kernel bandwidth) have been used in other studies of distance-dependent competition indices (e.g., Pukkala 1989; Kenkel et al. 1989; Rouvinen and Kuuluvainen 1997; Shi and Zhang 2003; Zhang and Shi 2004).

(2) Evaluation of Weighting Functions

1 In order to evaluate the spatial and spatial-attribute weighting functions, three trees were
2 randomly chosen from the three example plots (Figure 1 and Table 3). In general, the mean and
3 standard deviations of the weights obtained from the spatial-attribute weighing function were
4 smaller than those from the spatial weighting function. The small average of weights obtained
5 from the spatial-attribute weighting function ensures that the homoskedasticity of errors over all
6 predictor variables (e.g., DBH) for each focal tree. It is one of basic assumptions in linear
7 regression. Statistically, the spatial-attribute weights can improve estimation efficiency and
8 achieve unbiased estimation of the standard error of model parameters (Table 3). From the
9 biological point of view, it indicates a two-way competition during tree growth, meaning all trees
10 (large or small) compete for resources from the environment regardless of their sizes.

11 The BAGs predicted with the spatial-attribute weighting GWR model were closer to the
12 observed BAG than those predicted from the spatial weighting GWR model. For example, Tree
13 A was a large tree in the random plot (Figure 1b). Its nearer neighbors included large trees and
14 small trees within the kernel bandwidth of 7 m. Because the spatial-attribute weighting function
15 incorporated not only the tree sizes (competition) but also the geographical distances (spatial
16 impact) for the computation of weights, it resulted in better prediction for BAG. When the tree
17 DBH was used in the spatial-attribute weighting function, the predicted BAG was 0.0097,
18 however it was 0.0096 for the spatial weighting GWR model. Better predictions can also be
19 obtained with DBH^2 , APA and Hegyi's CI used in the spatial-attribute weighting function when
20 compared with the observed BAG (Table 3). Because the predicted BAGs were closer to the
21 observed BAG when the spatial-attribute weighting function was used than that obtained from
22 the spatial weighting function, the model residuals for this focal tree A were small.

1 The comparison among the predicted BAGs obtained from the spatial-attribute weights
2 indicated that APA and Hegyi's CI were better than DBH and DBH^2 (Table 3). For example, the
3 model residual was 0.00028, when DBH and DBH^2 were used in Equation [7] for the tree A in
4 the clustered plot. However, it was 0.00008 for APA and Hegyi's CI. These two competition
5 indices are the indicators of tree competition, which might be more accurately represent the local
6 condition, therefore better predictions can be gained.

7 For further comparing the difference between these two weighting functions, we took tree
8 A in the clustered plot as an example. The comparison of Figure 3a with Figure 3b, 3c, 3d, and
9 3e indicated that the weights obtained from the spatial weighting function was different from
10 these obtained from the spatial-attribute weighting function. The weights obtained with different
11 attributes were also different. The decay rates of the spatial-weighting function (Equation [6])
12 with the attributes of DBH^2 and Hegyi's CI were larger than these with DBH and APA (Figure
13 3). The spatial-attribute weights obtained with DBH and APA had a similar trend. Generally,
14 APA is positively correlated with DBH. In other words, large trees have large APA. Therefore
15 large trees can gain more space, light, and nutrient than small trees. Hegyi's CI has the opposite
16 meaning of APA. Large Hegyi's CI indicates the subject tree has strong competition from its
17 neighbors. In general, small trees have large Hegyi's CI. According to our comparison between
18 the spatial-attribute weights obtained using the competition indices, although they might be the
19 positively or negatively correlated with the tree size (i.e., DBH), they all followed the same
20 trend. The larger the difference in tree attributes (e.g., APA), the smaller the weight. The closer
21 the distance from the focal tree, the larger the weight.

22 **(3) Evaluation of Model Performance**

1 The GWR model was fit to the data of the three example plots. The predicted BAG was
2 obtained for each tree, and the RS, ARS, and RMSE were computed for each plot. The new
3 spatial-attribute weighting function always produced smaller RMSE than that of the spatial
4 weighting function, regardless of the spatial patterns of the example plots (Table 4). It implies
5 that the GWR model (Equation [2]) fits the data better if the spatial-attribute weighting function
6 is used.

7 In general, the GWR model with the spatial-attribute weighting function produces smaller
8 RS than that with the spatial weighting function (Figure 4). However, the difference between the
9 spatial weighting and spatial-attribute weighting functions is smaller for the regular plot (Figure
10 4a) than those for the random plot (Figure 4b) and clustered plot (Figure 4c). With the spatial
11 pattern from regularity to clustering, the model residuals obtained from the two weighting
12 functions become more dissimilar to each other. It indicates that the spatial pattern may have a
13 significant impact on the model performance. For the spatial-attribute weighting function,
14 different tree attributes resulted in different model residuals, however they were generally
15 smaller than that obtained from the spatial weighting function.

16 The absolute model residuals obtained from the spatial-attribute weighting function were
17 always smaller than that from the spatial weighting function (Figure 5). However, the absolute
18 model residuals have very similar patterns across the diameter classes between the two weighting
19 functions. The absolute model residual tends to decrease across the diameter classes for the
20 regular plot (Figure 5a), while it increases first from the diameter class 2 to 6 cm and then
21 decreases beyond 6 cm for the random plot (Figure 5b) and clustered plot (Figure 5c). The
22 absolute model residual with DBH^2 , APA and Hegyi's CI was generally smaller than that with
23 DBH.

1 Furthermore, if trees were not distributed evenly in the stand, the spatial-attribute weighting
2 GWR model would provide better predictions. With the development of Geographic Information
3 System (GIS), the GWR model can be easily incorporated into GIS to simulate forest dynamics
4 at the stand and landscape scales.

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6 **Acknowledgments:** The authors thank USDA Forest Service Northeastern Research Station NE-
7 4104 Group for providing AMORPHYS. Financial support was provided by the Michigan
8 Department of Natural Resources and a USDA National Research Initiative grant.

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1 Table 1. The initialization of the three example plots.

Spatial pattern	Tree Species	Number of Trees	Minimum DBH (cm)	Maximum DBH (cm)	Plot Size (m²)	Buffer Zone (m)
Regular	Loblolly Pine	100	7.5	20	10000	10
Random	Loblolly Pine	150	5.0	20	10000	10
Clustered	Loblolly Pine	200	2.5	20	10000	10

2 Note: DBH is the diameter at the breast height.

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1 Table 2. Descriptive statistics of individual tree measurements before and after the plot
 2 projection.

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Regular Plot					
Variable	N	Mean	Std Dev	Minimum	Maximum
DBH ^a (cm)	100	10.5982	2.7772	5.5300	16.9100
DBH ^b (cm)	95	21.7220	2.5856	14.8200	25.7500
BAG (m ² /tree)	95	0.0090	0.0014	0.0047	0.0106
Random Plot					
Variable	N	Mean	Std Dev	Minimum	Maximum
DBH ^a (cm)	150	8.7152	3.2013	2.9400	17.01
DBH ^b (cm)	142	19.2979	3.4833	11.8400	25.8600
BAG (m ² /tree)	142	0.0075	0.0020	0.0033	0.0105
Clustered Plot					
Variable	N	Mean	Std Dev	Minimum	Maximum
DBH ^a (cm)	200	7.1711	3.0235	2.5000	16.1600
DBH ^b (cm)	169	17.5802	3.4649	9.6600	25.0200
BAG (m ² /tree)	169	0.0065	0.0019	0.0023	0.0106

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5 Note:

6 ^a tree measurements before the plot projection.

7 ^b tree measurements after the plot projection.

8 BAG is the basal area growth during the 5-year growth period

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1 Table 3. Examples of the impact of the two weighting function on the model estimation with different spatial patterns.

		Spatial Weight						Spatial-attribute Weight									
		DBH			DBH ²			APA			Hegy's CI						
Plot	Tree	w_{ij}	w_{ij}	Predicted	w_{ij}	w_{ij}	Predicted	w_{ij}	w_{ij}	Predicted	w_{ij}	w_{ij}	Predicted	w_{ij}	w_{ij}	Predicted	Observed
		Mean	Std	BAG	Mean	Std	BAG	Mean	Std	BAG	Mean	Std	BAG	Mean	Std	BAG	BAG
Regular	A	0.0423	0.1629	0.0099	0.0234	0.1176	0.0099	0.0340	0.1489	0.0100	0.0249	0.1393	0.0099	0.0322	0.1499	0.0099	0.00999
	B	0.0255	0.1163	0.0094	0.0223	0.1128	0.0095	0.0198	0.1105	0.0091	0.0211	0.1105	0.0096	0.0162	0.1057	0.0095	0.00968
	C	0.0311	0.1304	0.0089	0.0368	0.1543	0.0089	0.0181	0.1111	0.0089	0.0256	0.1199	0.0089	0.0199	0.1113	0.0089	0.00886
Random	A	0.0129	0.0894	0.0096	0.0143	0.1052	0.0097	0.0088	0.0843	0.0101	0.0085	0.0845	0.0100	0.0071	0.0839	0.0101	0.01000
	B	0.0115	0.0899	0.0096	0.0094	0.0859	0.0095	0.0086	0.0853	0.0098	0.0091	0.0857	0.0098	0.0072	0.0839	0.0098	0.00950
	C	0.0146	0.0931	0.0077	0.0109	0.0884	0.0077	0.0099	0.0871	0.0076	0.0113	0.0890	0.0076	0.0108	0.0883	0.0076	0.00761
Clustered	A	0.0202	0.1145	0.0061	0.0221	0.1025	0.0062	0.0149	0.1035	0.0062	0.0157	0.1052	0.0064	0.0097	0.0826	0.0064	0.00648
	B	0.0172	0.1047	0.0075	0.0080	0.0799	0.0076	0.0123	0.0926	0.0076	0.0115	0.0865	0.0075	0.0159	0.1077	0.0081	0.00763
	C	0.0179	0.1032	0.0076	0.0118	0.0716	0.0076	0.0123	0.0881	0.0078	0.0129	0.0905	0.0076	0.0073	0.0785	0.0078	0.00787

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1 Table 4. The RMSE of the GWR models with the two weighting functions.

Weighting Function		RMSE		
		Regular Plot	Random Plot	Clustered Plot
Spatial weighting		0.000068	0.000125	0.000115
	DBH	0.000057	0.000105	0.000094
Spatial-attribute weighting	DBH ²	0.000048	0.000088	0.000114
	APA	0.000055	0.000083	0.000071
	Hegyí's CI	0.000042	0.000079	0.000078

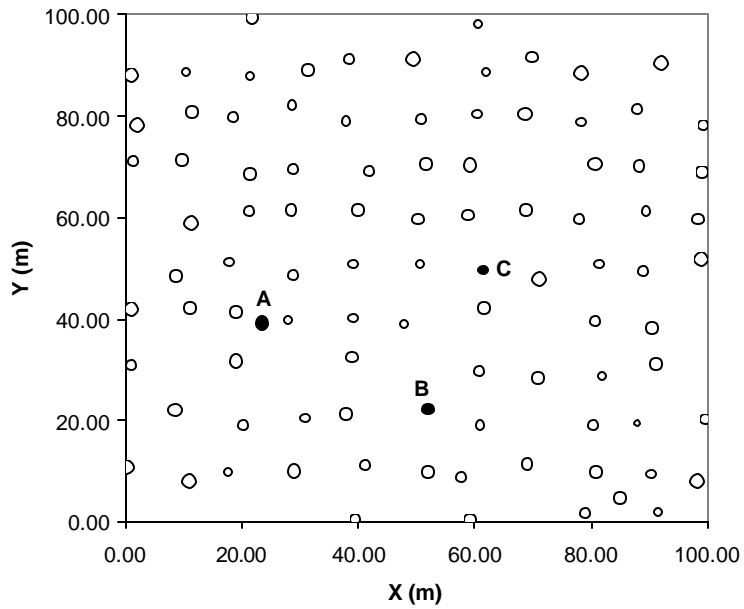
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1 Table 5. The paired t-tests for model residuals and absolute model residuals obtained from the
 2 GWR models with different weighting functions.

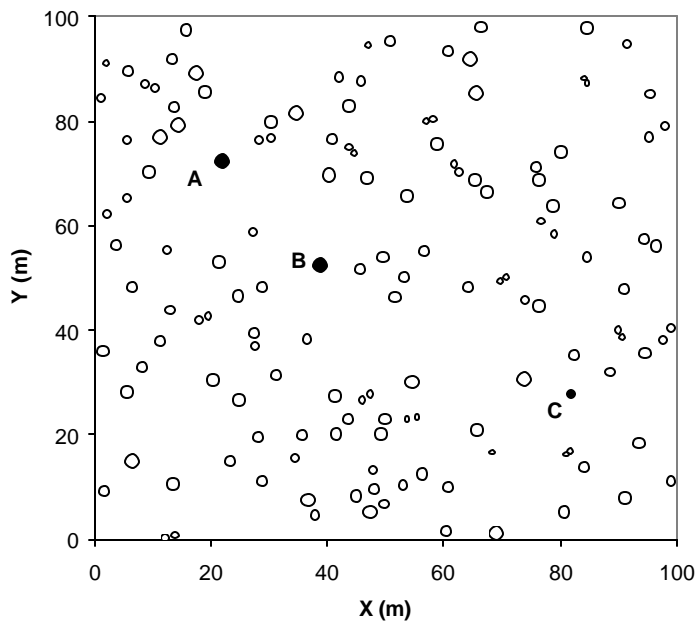
Plot	Attribute	Model Residuals	Model Absolute Residuals
		p-value	p-value
Regular	DBH	0.6850	<0.0001
	DBH ²	0.3019	<0.0001
	APA	0.7868	<0.0001
	Hegyí's CI	0.5036	<0.0001
Random	DBH	0.5243	<0.0001
	DBH ²	0.7046	<0.0001
	APA	0.3295	<0.0001
	Hegyí's CI	0.0638	<0.0001
Clustered	DBH	0.8383	<0.0001
	DBH ²	0.1525	0.0003
	APA	0.1525	<0.0001
	Hegyí's CI	0.0611	<0.0001

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1 **Figure 1.** Map of tree locations for the three example plots: (a).Regularity; (b). Randomness; and
2 (c). Clustering. (The circle is proportional to the tree DBH).
3 a.

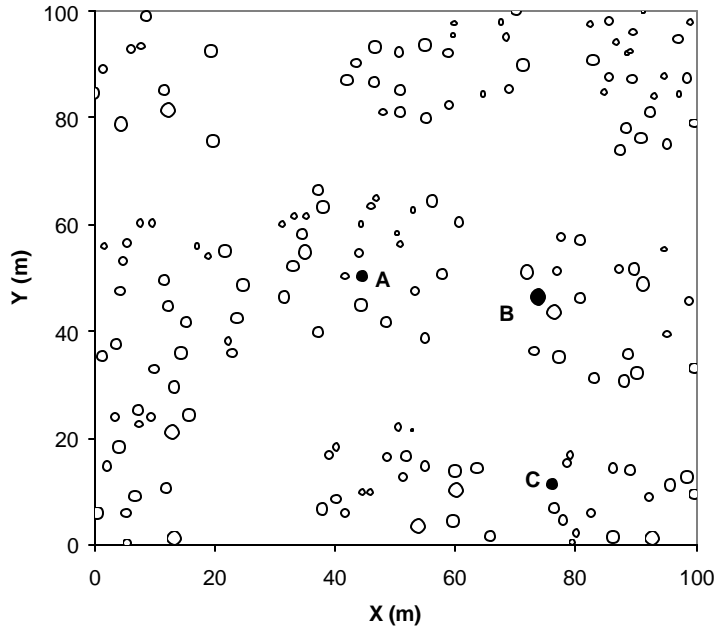


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1 c.



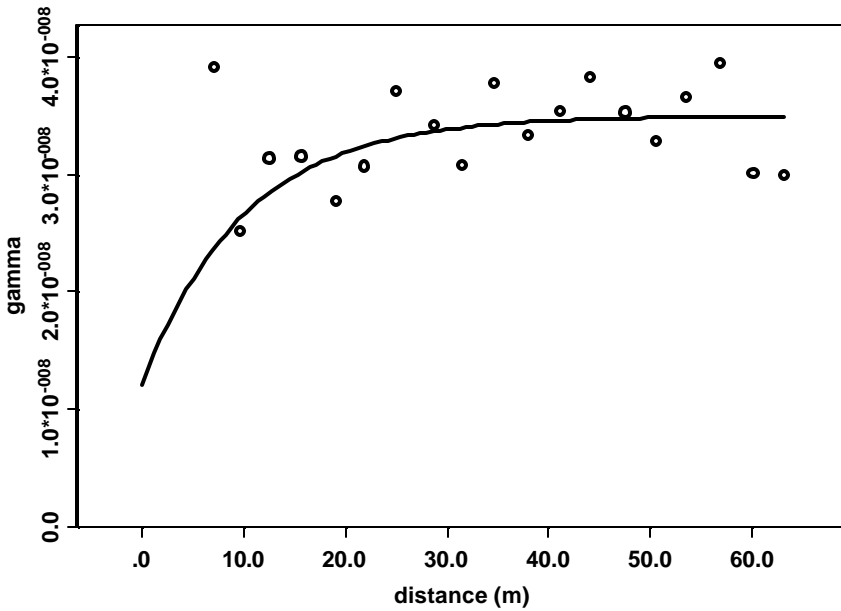
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4 **Figure 2.** Variogram of the OLS model residuals: (a). Regularity, (b). Randomness, (c).

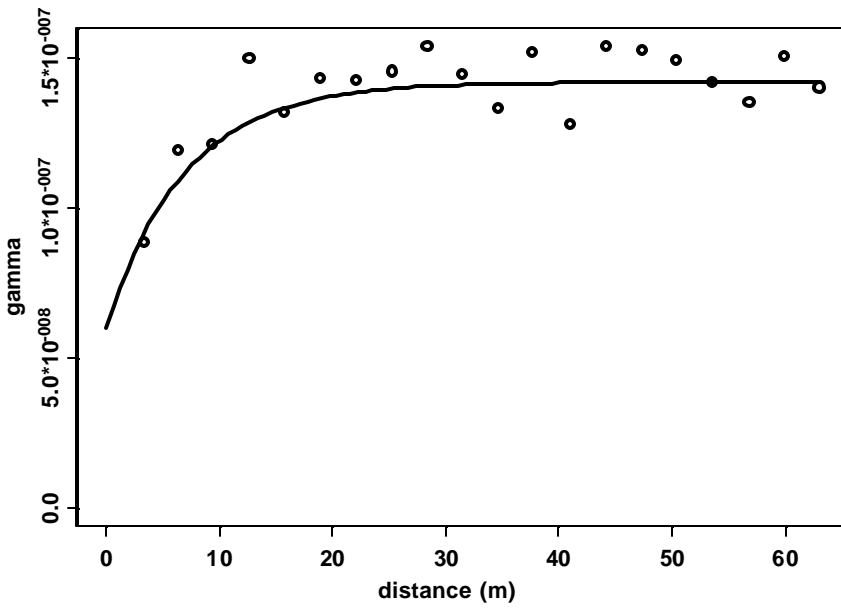
5 Clustering.

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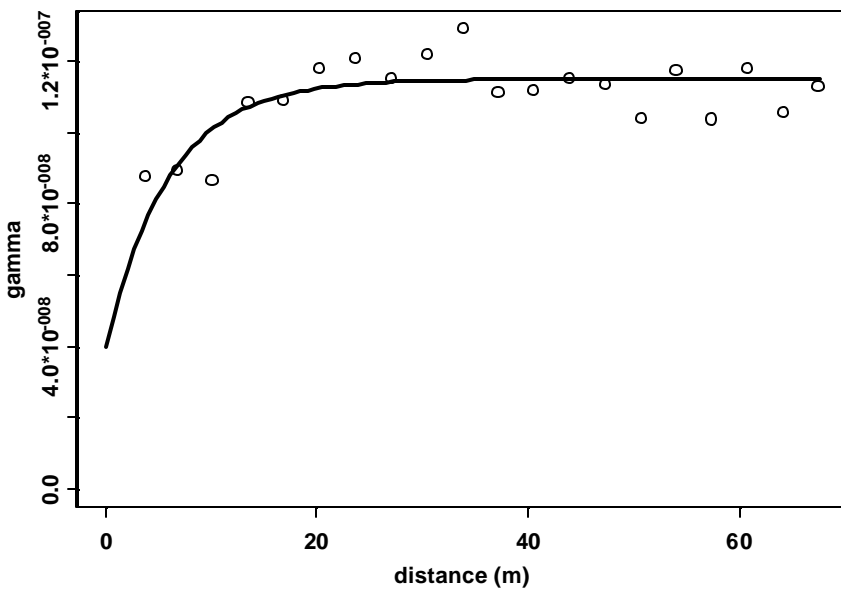
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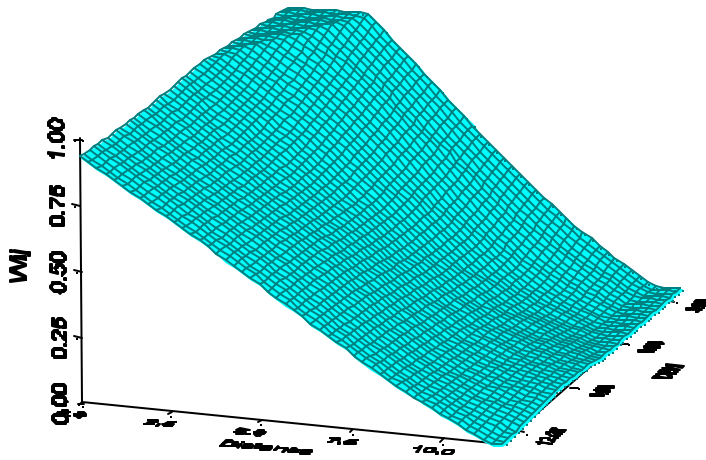
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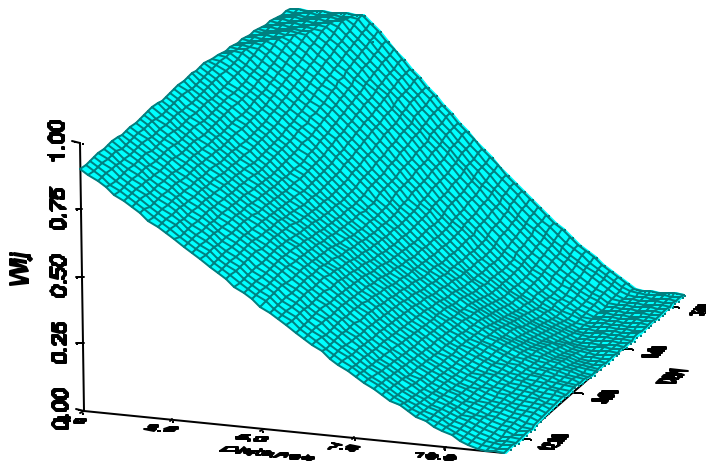
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1 **Figure 3.** An example of spatial weight and spatial-attribute weight with different tree attributes
 2 (i.e., DBH, Hegyi's Ci, APA, and DBH²): tree A in the clustered plot (Note, for other trees in the
 3 regular, random and clustered plots, similar results can be obtained). Distance= the distance (m)
 4 between the subject tree and its neighboring trees. W_{ij}=spatial-attribute weight. DBH=diameter
 5 at breast height (cm)).
 6 a. Spatial weight; b. Spatial-attribute weight with DBH; c. Spatial-attribute weight with DBH²; d.
 7 Spatial-attribute weight with APA; e. Spatial-attribute weight with Hegyi's CI.

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 9 a. Spatial weight
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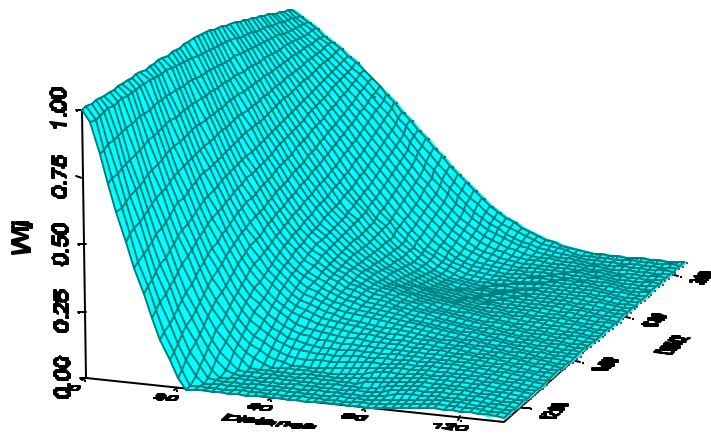


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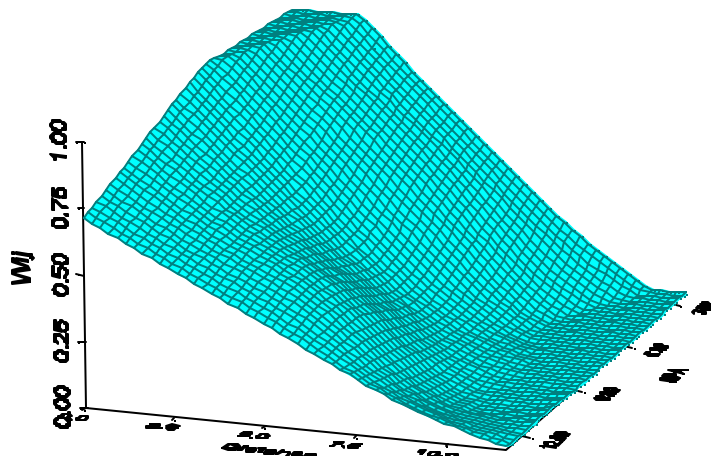


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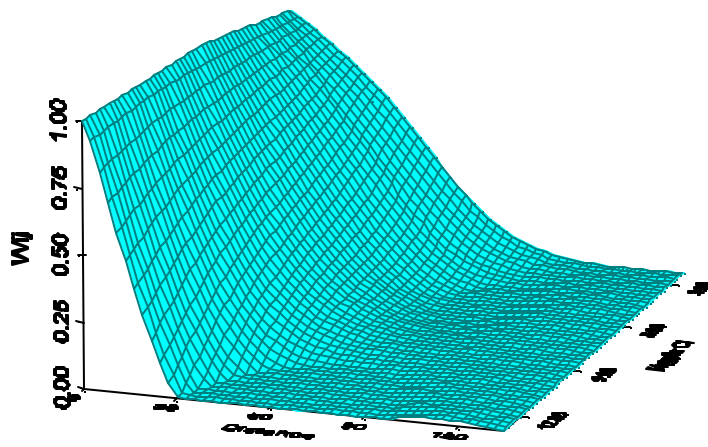


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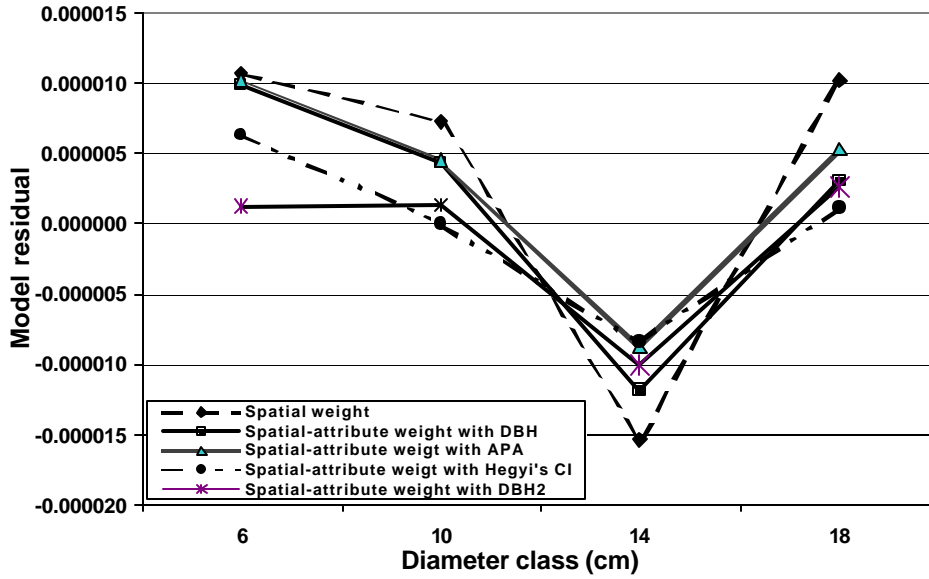
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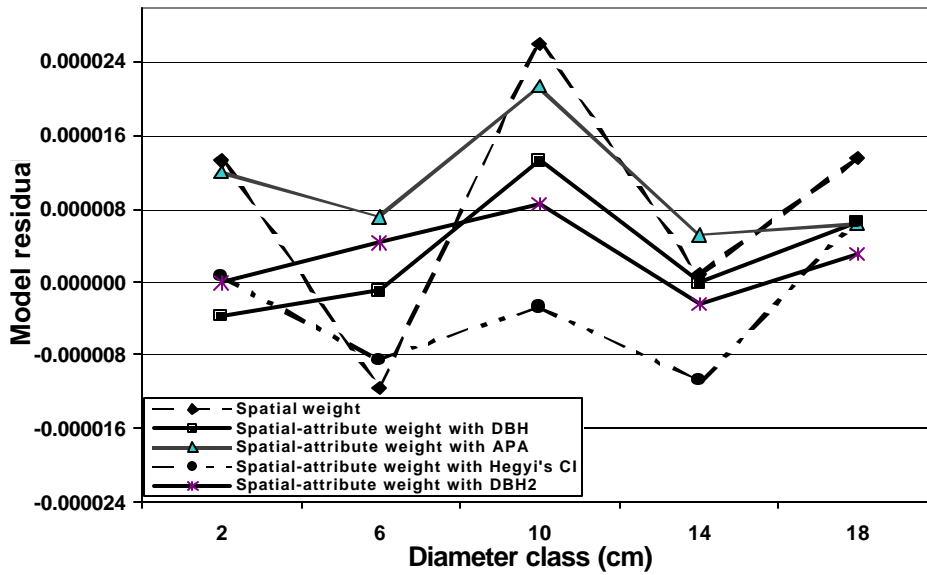


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1 **Figure 4.** Model residuals across the diameter classes: (a). Regularity; (b). Randomness; and (c).
 2 Clustering.
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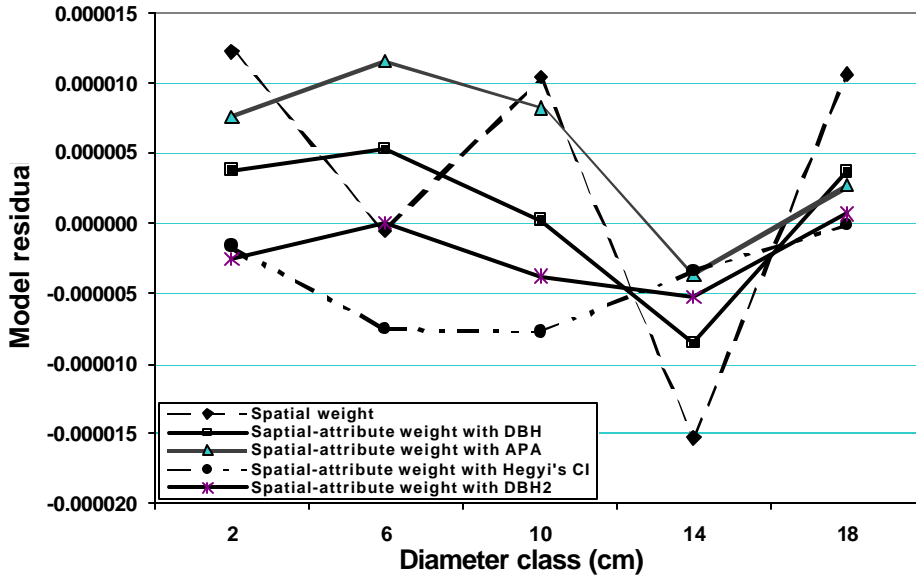
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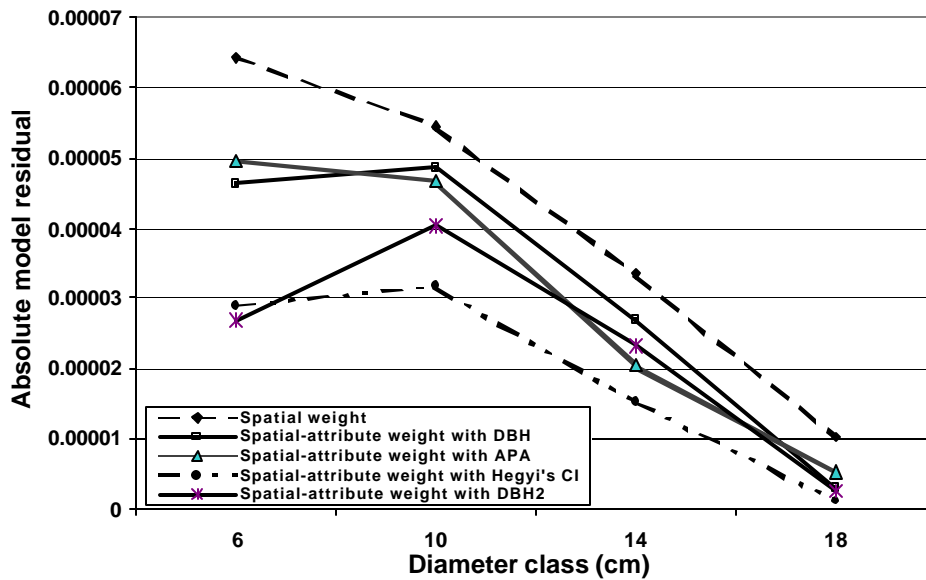
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4 **Figure 5.** Absolute model residuals across the diameter classes: (a). Regularity; (b).
5 Randomness; and (c). Clustering.

6 a.



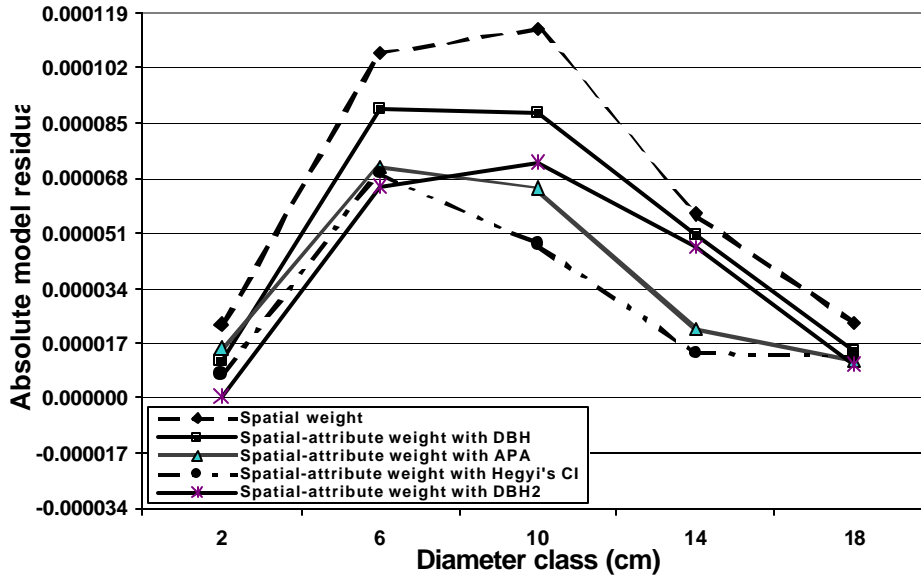
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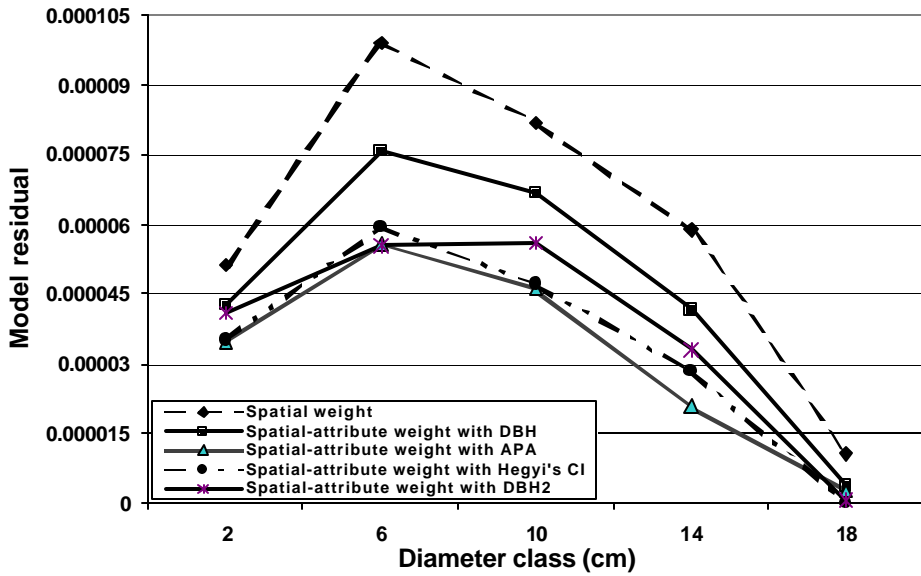
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3 b.



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