Incorporating both Geographical Space and Attribute Space into the Kernel Weighting Function of Geographically Weighted Regression

Haijin Shi, Lianjun Zhang, and Jianguo Liu

Haijin Shi (corresponding author), Research Associate, Phone: (517) 432-8256, Fax: (517) 432-1699, E-mail: hshi@msu.edu, Department of Fisheries and Wildlife, 13 Natural Resources Building, Michigan State University, East Lansing, MI. 48824.

Lianjun Zhang, Professor, Phone: (315) 470-6558, Fax: (315) 470-6535, E-mail: lizhang@esf.edu, Faculty of Forest and Natural Resources Management, State University of New York, College of Environmental Science and Forestry, One Forestry Drive, Syracuse, NY 13210.

Jianguo Liu, Rachel Carson Chair, Professor, Phone: (517) 355-1810, Fax: (517) 432-1699, jliu@panda.msu.edu, Department of Fisheries and Wildlife, 13 Natural Resources Building, Michigan State University, East Lansing, MI. 48824.

Abstract

Geographically Weighted Regression (GWR), following the general principle of local smoothing and locally weighted regression, has been developed to study the spatial heterogeneity in a regression context. The kernel weighting function is the key component used to account for the spatial heterogeneity in GWR. The spatial heterogeneity generally results from both "geographical space" and "attribute space". However the current kernel weighting function only considers the geographical distances of the neighbors from a focal point in the study area, while the attributes of the focal point and its neighbors are totally ignored. In this study, we proposed a new kernel weighting function that combines the "geographical space" and "attribute space" between the focal point and its neighbors such that (1) neighbors with greater geographical distances from the focal point are assigned smaller weights, and (2) at a given geographical distance, neighbors with similar sizes to the focal point are assigned larger weights. The characteristics of the new weighting function are investigated with four tree attributes, diameter at breast height (DBH), DBH^2, area potentially available (APA) and Hegyi's competition index (CI), and the new weighting function is also tested using three simulated forest stands with different spatial patterns. The results indicate that smaller model residuals and better predictions can be obtained from the GWR model with the new spatial-attribute kernel weighting function than that from the traditional GWR model.

Key words: Geographically weighted regression, spatial pattern analysis, spatial heterogeneity, locally weighted regression.

Introduction

Local modeling has become an increased interest in recent years. A number of approaches, such as spline functions and kernel regression, have been developed for examining local relationships in nonspatial data (Wahba 1990; Green and Silverman 1994; Cleveland and Devlin 1988). In spatial data, geographically weighted regression (GWR) has become popular for depicting the spatial heterogeneity in a regression context in recent years (Brundson et al. 1996; Fotheringham et al. 2002; Zhang and Shi 2004). In GWR, any spatial heterogeneity in the relationship is accounted for by the local estimation of model coefficients through a spatial weighting function. This spatial weighting function is a decreasing function of distance (geographical space) from the focal observation (x_0) so that the impact of the neighbors (x_i, i=1…k, k is the number of neighbors) nearby is stronger than those farther away. In general, spatial data consist of both attribute and spatial information (e.g., spatial coordinates; Fotheringham et al. 2002). GWR uses only the distance (geographical space) to determine the weights. It may not be realistic and reasonable, because the effects of the attributes of the focal observation and its neighbors are totally ignored. In other words, no matter how large or small of the attributes of its neighbors, if they have the same geographical distance from the focal observation, they would have the same weight. Apparently, the attribute information should be considered in the weighting function.

The development of GWR follows the general principle of local smoothing and locally weighted regression (Leung et al 2000; Páez et al 2002), in which the weights are determined by the size of the residuals (Cleveland 1979; Cleveland and Devlin 1988; Casetti 1982; Casetti and Can 1999). For a given focal point x_0 in the locally weighted regression, if the sizes of its neighbors x_i are similar to the size of x_0, the "distance of attribute space" between x_0 and x_i is small. These neighbors are assigned large weights by the weighting function. In contrast, its neighbors x_i, with the sizes dissimilar to the size of the x_0, are assigned small weights because they are far away from x_0 in the "distance of attribute space". In other words, the weights are determined by the "attribute space" instead of the "geographical space" (Leung et al. 2000). This approach pays more attention to the fitting of the dependent variable rather than on spatially varying parameters. However, the "attribute space" approach does not consider geographical locations of the neighbors and the relative distance (geographical space) between x_0 and x_i.

In this study, we propose a new approach that will incorporate the attribute of the observations into the spatial weighting function used in GWR. The new weighting function will combine the "geographical space" and "attribute space" between the focal point (x_0) and the neighbors (x_i) such that (1) the neighbors (x_i) with large geographical distances from x_0 will be assigned small weights, and *vice versa*, and (2) at a given geographical distance, the neighbors (x_i) with similar attributes to x_0 will be assigned large weights, and *vice versa*. The properties of the "spatial-attribute" weighting function were tested with simulated forest stands (see Data section) with regard to spatial continuity as well as statistical and biological interpretation.

One attribute and three attribute functions were used once at a time in the spatial-attribute weighting function. The attribute used in this study is the tree diameter at breast height (DBH). The three attribute functions are DBH^2 and two traditional competition indices (CI), Hegyi's CI (Hegyi 1974) and area potentially available (APA; Brown 1965; Moore et al 1973). In general, the tree attribute is defined as the measurable tree characteristics (e.g., DBH). However, for simplicity, we defined these three attribute functions as tree attributes in this study. DBH^2 is proportional to the tree basal area. The selection of these attributes is to test whether different tree characteristics can alter the model performance. The reason of choosing APA and Hegyi's

CI is that they have been widely used in forest growth and yield models (Newton and Jolliffe 1998; Shi and Zhang 2003), and demonstrated that they are useful indices for measuring tree competition (Moore et al. 1973; Biging and Dobbertin 1995). Another reason is that a large Hegyi's CI means the subject tree has relatively strong competition from its neighbors, however a large APA indicates that the subject tree has stronger competition than its neighbors. Therefore, these two CIs have opposite meaning for interpreting tree competition. It is useful to test the performance of the spatial-attribute weighting function.

The objectives of this study were (1) to generate three example plots with different spatial patterns (i.e. regular, random, and clustered) of tree locations, (2) to model the relationship between tree size and growth using the GWR methodology with different weighting functions (i.e. spatial weighting and spatial-attribute weighting functions), and (3) to compare and evaluate the performance of the two weighting functions for modeling the effects of spatial heterogeneity on tree growth.

Data

Three example plots used in this study were generated using a process-based stand model AMORPHYS (Valentine et al. 2000). This public-domain software is developed by USDA Forest Service Northeastern Research Station and can be downloaded from the web site "http://ftp.fs.fed.us/ne/durham/4104/products/InstallAMORPHYS.ex_". AMORPHYS can (1) generate the locations of model trees, (2) sample tree diameters from a target distribution (e.g., a two-parameter Weibull) and assign those diameters to model-tree locations, (3) compute other tree attributes such as height and crown length, and (4) predict tree growth (Valentine et al. 2000). The generation of tree locations (coordinates) is based on an algorithm called LPOINT (Penridge 1986) that can produce two-dimensional point patterns from regular, through Poisson or random, to strongly clumped, depending on the values of two parameters, ? and ?. ? is the randomness parameter and ? is the mean point density (for detailed information, see Penridge 1986).

We generated an example plot for each of three spatial patterns, i.e. regularity, randomness, and clustering. Each example plot was a 100 x 100 m square plot. The initialization of the plots was manually set up for the model (Table 1). The specifications of the model parameters were (1) regular plot: ? = 0.45 and ? = 1.0, (2) random plot: ? = 1.0 and ? = 1.0, and (3) cluster plot: ? = 10.0 and ? = 5.0. Figure 1 shows the map of tree locations for the three example plots. Then tree initial diameters were obtained from a Weibull distribution and were assigned to the tree locations. The trees in the three plots were projected for a growth period of 5 years. The descriptive statistics of tree initial diameter at breast height (DBH) and 5-year basal area growth (BAG) were listed in Table 2.

Methods

(1) GWR Model

Suppose we have a set of n observations $\{X_{ij}\}$ with the spatial coordinates $\{(u_i, v_i)\}$, $i = 1, 2, \ldots, n$, on p independent or predictor variables, $j = 1, 2, \ldots, p$, and a set of n observations on a dependent or response variable $\{y_i\}$. The underlying model for GWR is

$$y_i = b_0(u_i, v_i) + \sum_{j=1}^{p} X_{ij}\, b_j(u_i, v_i) + e_i \qquad [2]$$

where $\{ß_0(u_i, v_i), ß_1(u_i, v_i), \ldots, ß_p(u_i, v_i)\}$ are p+1 continuous functions of the location (u_i, v_i) in the study area. The e_i is the random error term with a distribution $N(0, s^2 I)$. Suppose $W_i(u_i, v_i)$ is

$$W_i(u_i, v_i) = \begin{pmatrix} w_{i1} & 0 & \cdots & 0 \\ 0 & w_{i2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & w_{in} \end{pmatrix} \qquad [3]$$

The estimator of *ß*$_i$ is given by GWR:

$$\hat{b}_i = \left(X^T W_i(u_i, v_i) X\right)^{-1} X^T W_i(u_i, v_i)\, y \qquad [4]$$

After the GWR model regression, a set of parameter estimates can be obtained for each data point. The weights (w_{ij}) in the weight matrix $W_i(u_i, v_i)$ is a decreasing function of distance d_{ij} between subject i and its neighboring location j. In general, the spatial weighting function is taken as the exponential distance-decay form:

$$w_{ij} = \exp(-\frac{d_{ij}^2}{h^2}) \qquad [5]$$

where h is called kernel bandwidth. If locations i and j coincide (i.e., $d_{ij} = 0$), w_{ij} equals one; while w_{ij} decreases according to a Gaussian curve as the distance d_{ij} increases. However, the weights are nonzero for all data points, no matter how far they are from the center i (Fotheringham et al. 2002). The kernel bandwidth can be determined by (1) predefined kernel bandwidth; (2) Minimum Akaike Information Criterion (AIC); (3) cross-validation procedure; (Fotheringham et al 2002; Zhang and Shi 2004).

(2) Spatial-Attribute Weighting Function

The spatial weighting function (Equation [5]) only takes the geographical distance into account, and ignores the influence of the observation's attributes. Since both tree size and location have strong impacts on competition among trees, crown structure, growth, and mortality (e.g., Miller and Weiner 1989; Moeur 1993; Newton and Jollife 1998), we propose to modify Equation [5] as follows:

$$w_{ij} = \exp\left(-\frac{d_{ij}^2}{h^2} \times f(t)\right) \qquad [6]$$

where $f(t)$ is a function that changes the weight w_{ij} according to the difference (t) between the size of the focal tree and its neighbors.

Following the idea of the weighting function in locally weighted regression techniques (Cleveland 1979; Castti 1982; Cleveland and Devlin 1988), the weight should decrease as the difference between the focal observation (i) and its neighbors (j) increases. The symmetric weight is one of the important properties of the weighting function because it reduces bias (Cleveland and Devlin 1988). The $f(t)$ function can be bisquare, "tribcube", or exponential functions, therefore we propose the following format for the $f(t)$ function:

$$f(t) = \exp\left(\left|1 - \frac{ATT_{ij}}{ATT_{ii}}\right|\right) \qquad [7]$$

where ATT_{ii} is the attribute of the focal tree i, and ATT_{ij} is the same attribute of the neighboring tree j. In this study, the tree attributes include DBH, DBH^2, Hegyi's CI and APA. APA requires construction of Voronoi polygons (via the Dirichlet tessellation) around each tree. The distance between each tree is bisected at right angles, and these successive right angle lines are joined to form a polygon. Hegyi's CI is defined as:

$$Hegyi'\,CI = \sum_{j}^{n} (c(h) DBH_j / DBH_i) / L_{ij} \qquad [8]$$

where i denotes the subject tree, j denotes the neighboring competitor, DBH is the diameter at breast height, $c_{ij}(d)$ is a spatial matrix for a given bandwidth (h), n is the number of neighbors inside a neighborhood zone of the subject tree, L_{ij} is the distance between tree i and tree j.

The spatial-attribute weighting function (Equation [6]) takes into account both the geographical distance and the attribute of the focal tree and its neighbors. If the tree attributes (e.g., DBH) of the neighboring trees are greatly different from a given focal tree, the smaller weights are assigned to these neighboring trees. In contrast, if the differences of tree attributes between the focal tree and its neighbors are small, larger weights are assigned according to Equation [6]. In the case that the size of a neighboring tree is the same as the focal tree (i.e., $f(t)=1$), the weight for that tree is determined by the spatial distance only. Biologically, it implies that the competition is a reciprocal process. Large trees have influence on small trees, however small trees also compete for resources with large trees.

(3) Regression Model

Many models for tree diameter or basal area growth have been developed over the past several decades. Different models work well under different conditions (Vanclay 1994). In this study, we chose the following linear regression model to investigate the differences between the spatial weighting function (Equation [5]) and the spatial-attribute weighting function (Equation [6]) used in the GWR model.

$$\log\left(BAG + 1\right) = b_0(u,v) + b_1(u,v)\cdot\log\left(DBH\right) + b_2(u,v)\cdot DBH^2 + e \quad [8]$$

where *BAG* is the basal area growth, *DBH* is the initial tree DBH, log is a 10-based logarithm, $\beta_0(u, v)$~ $\beta_2(u, v)$ are regression coefficients to be estimated, and e is the model random error. If the spatial coordinates are removed from the above model, Equation [8] becomes the derivative model of the Bertalanffy growth function, which has been used as a basic function in several forest growth and yield models due to its simplicity and robust predictions (e.g., Wykoff 1990; Hann and Larsen 1991; Vanclay 1994; Monserud and Sterba 1996).

(4) Model Comparison and Evaluation

The GWR models with the spatial and spatial-attribute weighting functions were compared based on the criteria of the root mean square error (RMSE) and bias (Maltamo et al. 1995) such as

$$RSME = \sqrt{\frac{\sum_i (BAG_i - B\hat{AG}_i)^2}{n}} \quad [9]$$

where n is the number of observations, BAG_i is the basal area growth (BAG) of the ith tree, $B\hat{AG}_i$ is the predicted BAG of the ith tree.

Model residual (RS = observed - predicted) and absolute model residual (ARE = |observed - predicted|) were also evaluated across diameter classes. Then paired t-tests were used to compare the differences of RS and ARE between the GWR models with the two weighting functions. Finally, the contour plots of the parameter estimates from the two weighting functions were mapped for spatial assessment.

Results

(1) Determination of Kernel Bandwidth

In this study, we decided to predefine the bandwidth for the three example plots based on the variogram of the OLS model residuals. Because of the relatively small size of the example plot, reasonable kernel bandwidth can not be obtained using the cross-validation procedure and minimum AIC approach. The variogram curves indicate that the ranges of these variograms (Figure 2) for the three example plots were 10m (regular plot), 7m (random plot), and 6m (clustered plot), respectively. Since the range of a variogram indicates that there was no spatial autocorrelation between trees beyond the distance (Isaaks and Srivastava 1989; Kohl and Gertner 1997), we chose these three range distances as the bandwidth for the three example plots. In addition, similar distances (or kernel bandwidth) have been used in other studies of distance-dependent competition indices (e.g., Pukkala 1989; Kenkel et al. 1989; Rouvinen and Kuuluvainen 1997; Shi and Zhang 2003; Zhang and Shi 2004).

(2) Evaluation of Weighting Functions

In order to evaluate the spatial and spatial-attribute weighting functions, three trees were randomly chosen from the three example plots (Figure 1 and Table 3). In general, the mean and standard deviations of the weights obtained from the spatial-attribute weighing function were smaller than those from the spatial weighting function. The small average of weights obtained from the spatial-attribute weighting function ensures that the homeskedasticity of errors over all predictor variables (e.g., DBH) for each focal tree. It is one of basic assumptions in linear regression. Statistically, the spatial-attribute weights can improve estimation efficiency and achieve unbiased estimation of the standard error of model parameters (Table 3). From the biological point of view, it indicates a two-way competition during tree growth, meaning all trees (large or small) compete for resources from the environment regardless of their sizes.

The BAGs predicted with the spatial-attribute weighting GWR model were closer to the observed BAG than those predicted from the spatial weighting GWR model. For example, Tree A was a large tree in the random plot (Figure 1b). Its nearer neighbors included large trees and small trees within the kernel bandwidth of 7 m. Because the spatial-attribute weighting function incorporated not only the tree sizes (competition) but also the geographical distances (spatial impact) for the computation of weights, it resulted in better prediction for BAG. When the tree DBH was used in the spatial-attribute weighting function, the predicted BAG was 0.0097, however it was 0.0096 for the spatial weighting GWR model. Better predictions can also be obtained with DBH^2, APA and Hegyi's CI used in the spatial-attribute weighting function when compared with the observed BAG (Table 3). Because the predicted BAGs were closer to the observed BAG when the spatial-attribute weighting function was used than that obtained from the spatial weighting function, the model residuals for this focal tree A were small.

The comparison among the predicted BAGs obtained from the spatial-attribute weights indicated that APA and Hegyi's CI were better than DBH and DBH^2 (Table 3). For example, the model residual was 0.00028, when DBH and DBH^2 were used in Equation [7] for the tree A in the clustered plot. However, it was 0.00008 for APA and Hegyi's CI. These two competition indices are the indicators of tree competition, which might be more accurately represent the local condition, therefore better predictions can be gained.

For further comparing the difference between these two weighting functions, we took tree A in the clustered plot as an example. The comparison of Figure 3a with Figure 3b, 3c, 3d, and 3e indicated that the weights obtained from the spatial weighting function was different from these obtained from the spatial-attribute weighting function. The weights obtained with different attributes were also different. The decay rates of the spatial-weighting function (Equation [6]) with the attributes of DBH^2 and Hegyi's CI were larger than these with DBH and APA (Figure 3). The spatial-attribute weights obtained with DBH and APA had a similar trend. Generally, APA is positively correlated with DBH. In other words, large trees have large APA. Therefore large trees can gain more space, light, and nutrient than small trees. Hegyi's CI has the opposite meaning of APA. Large Hegyi's CI indicates the subject tree has strong competition from its neighbors. In general, small trees have large Hegyi's CI. According to our comparison between the spatial-attribute weights obtained using the competition indices, although they might be the positively or negatively correlated with the tree size (i.e., DBH), they all followed the same trend. The larger the difference in tree attributes (e.g., APA), the smaller the weight. The closer the distance from the focal tree, the larger the weight.

(3) Evaluation of Model Performance

The GWR model was fit to the data of the three example plots. The predicted BAG was obtained for each tree, and the RS, ARS, and RMSE were computed for each plot. The new spatial-attribute weighting function always produced smaller RMSE than that of the spatial weighting function, regardless of the spatial patterns of the example plots (Table 4). It implies that the GWR model (Equation [2]) fits the data better if the spatial-attribute weighting function is used.

In general, the GWR model with the spatial-attribute weighting function produces smaller RS than that with the spatial weighting function (Figure 4). However, the difference between the spatial weighting and spatial-attribute weighting functions is smaller for the regular plot (Figure 4a) than those for the random plot (Figure 4b) and clustered plot (Figure 4c). With the spatial pattern from regularity to clustering, the model residuals obtained from the two weighting functions become more dissimilar to each other. It indicates that the spatial pattern may have a significant impact on the model performance. For the spatial-attribute weighting function, different tree attributes resulted in different model residuals, however they were generally smaller than that obtained from the spatial weighting function.

The absolute model residuals obtained from the spatial-attribute weighting function were always smaller than that from the spatial weighting function (Figure 5). However, the absolute model residuals have very similar patterns across the diameter classes between the two weighting functions. The absolute model residual tends to decrease across the diameter classes for the regular plot (Figure 5a), while it increases first from the diameter class 2 to 6 cm and then decreases beyond 6 cm for the random plot (Figure 5b) and clustered plot (Figure 5c). The absolute model residual with DBH^2, APA and Hegyi's CI was generally smaller than that with DBH.

The results of the paired t-tests indicate that there are no significant differences in model residuals between the spatial and spatial-attribute weighting GWR models (Table 5). It is expected because the mean of the residuals from an unbiased model should be close to zero; consequently the difference between the means from the two GWR models should also be close to zero. On the other hand, the absolute model residuals represent the magnitudes of the residuals regardless of whether they are positive or negative. The paired t-tests for the absolute model residuals indicate that there are significant differences between the two weighting functions, and the GWR model with the spatial-attribute weighting function produces model errors smaller in magnitude than the one with the spatial weighting function (Table 5).

Conclusions

In general, the spatial-attribute weighting function performs better than the spatial weighting function using the simulated forest stands. Similar results can be obtained with other spatial data due to the mathematical properties embedded in Equation [6] and [7]. By improving the model fitting by the spatial-attribute weighing method, we can obtain a more accurate prediction of forest growth and yield. Forest researchers can use it to investigate the complex relationships among forest competition and growth incorporating spatial variation (Zhang and Shi 2004).

The GWR method is a useful tool to investigate spatial heterogeneity according to the analysis of the model fitting with our simulated example plots (i.e. regular, random, and clustered plots). Not only the spatial information but also the attribute of trees can be incorporated into the weighting function of the GWR model. Our results indicate that no matter what spatial patterns existed in the stand or what attributes were used, the performance of the GWR model with the spatial-attribute weighting method would be better than that without it.

Furthermore, if trees were not distributed evenly in the stand, the spatial-attribute weighting GWR model would provide better predictions. With the development of Geographic Information System (GIS), the GWR model can be easily incorporated into GIS to simulate forest dynamics at the stand and landscape scales.

Acknowledgments: The authors thank USDA Forest Service Northeastern Research Station NE-4104 Group for providing AMORPHYS. Financial support was provided by the Michigan Department of Natural Resources and a USDA National Research Initiative grant.

References

Anselin, L. 1988. Spatial Econometrics: Methods and Models. Dordrecht: Kluwer Academic Publishers. 284p.

Anselin, L. 1990. What is special about spatial data? Alternative perspectives on spatial data analysis. P 63-77 *in* Spatial Statistics: Past, Present, and Future. D. Griffith (ed.). Ann Arbor, Michigan: Institute of Mathematical Geography.

Anselin, L. 1993. Discrete space autoregressive models. P 454-469. *in* Environmental Modeling with GIS. Ed. Goodchild, M.F., B.O. Parks, and T. Steyaert. Oxford: Oxford University Press.

Anselin, L. and Griffith, D.A. 1988. Do spatial effects really matter in regression analysis. Papers of the Regional Science Association 65:11-34.

Brunsdon, C.A., Fotheringham, A.S. and Charlton, M.E. 1996. Geographically weighted regression: A method for exploring spatial nonstationarity. Geographical Analysis 28(4):281-298..

Brunsdon, C.A., Fotheringham, A.S. and Charlton, M.E. 1998. Geographically weighted regression – modeling spatial non-stationary. The Statistician 47(3):431-443.

Casetti, E. 1972. Generating models by the expansion method: Application to geographic research. Geog. Anal. 4:81-91.

Casetti, E. 1982. Drift analysis of regression parameters: an application to the investigation of fertility development relations. Model. and Simul. 13: 961-966.

Casetti, E. 1997. The expansion method, mathematical modeling, and spatial econometrics. Inter. Reg. Sci. Rev. 20:9-32.

Casetti, E. and Can, A. 1999. The economic estimation and testing of DARP models. J. Geog. Syst. 1: 91-106.

Cleveland, W. 1979. Robust locally weighted regression and smoothing scatterplots. JASA. 74:829-836.

Cleveland, W.S. and Devlin, S.J. 1988. Locally weighted regression: an approach to regression analysis by local fitting. JASA. 83: 596-610.

Diggle, P.J. 1983. Statistical Analysis of Spatial Point Patterns. Academic. Landon. 148p.

Dovciak, M., Frelich, L.E. and Reich, P.B. 2001. Discordance in spatial patterns of white pine (*Pinus strobus*) size-classes in a patchy near-boreal forest. J. Ecol. 89:280-291.

Fotheringham, A.S. 1997. Trends in quantitative methods I: Stressing the local. Prog. in Human Geog. 21:88-96.

Fotheringham, A.S. and Brunsdon, C.. 1999. Local forms of spatial analysis. Geog. Anal. 31:340-358.

Fotheringham, A.S., Brunsdon, C. and Charlton, M. 2000. Quantitative Geography: Perspectives on Spatial Data Analysis. SAGE Publications. 270p.

Fotheringham, A.S., Brunsdon, C. and Charlton, M.E. 2002. Geographically Weighted Regression: The Analysis of Spatially Varying Relationships, Chichester: Wiley.

Fox, J.C., Ades, P.K. and Bi, H. 2001. Stochastic structure and individual-tree growth models. For. Ecol. Manage. 154:261-276.

Frohlich, M. and Quednau, H.D. 1995. Statistical analysis of the distribution pattern of natural regeneration in forests. For. Ecol. Manage. 73:45-57.

Garcia, O. 1994. The state-space approach in growth modeling. Can. J. For. Res. 24:1894-1903.

Gorr, W.L. and Olligschlaeger, A.M. 1994. Weighted spatial adaptive filtering: Monte Carlo studies and application to Illicit Drub Market Modeling. Geog. Anal. 26(1):67-87

Green, P.J., and Silverman B.W., 1994. Nonparametric regression and generalized linear models: roughness penalty approach. London, Chapman & Hall.

Haase, P. 1995. Spatial pattern analysis in ecology based on Ripley's K-function: Introduction and methods of edge correction. J. Veget. Sci. 6:575-582.

Haining, R. 1978. Estimating spatial interaction models. Environment and Planning A 10:305-320.

Hann, D.W.. and Larsen, D.R. 1991. Diameter growth equations for fourteen tree species in Southwest Oregon. Forest Research Lab., Oregon State University, Corvallis, Oregon. Research Bulletin 69. 18 p.

Hegyi, F. 1974. A simulation for managing jack-pine stands. P. 74-90 *in* Growth Models for tree and stand simulation, Fries, J. (ed.). Roy. Coll. of For., Stockholm, Sweden.

Isaaks, E.H.. and Srivastava, R.M. 1989. An Introduction to Applied Geostatistics. Oxford University Press. New York. 561 p.

Jones, J. and Casetti, E. 1992. Applications of the expansion method Routledge, London, 375p.

Kenkel, N.C. 1988. Pattern of self-thinning in jack pine: testing the random mortality hypothesis. Ecology 69:1017-1024.

Kenkel, N.C., Hoskins, J.A. and Hoskins, W.D. 1989. Local competition in a naturally established jack pine stand. Can. J. Bot. 67:2630-2635.

Kohl, M., and Gertner, G. 1997. Geostatistics in evaluating forest damage surveys: considerations on methods for describing spatial distributions. For. Ecol. Manag. 95:131-140.

Lee, J. and Wong, D.W.S. 2001. Statistical analysis with ArcView GIS. John Wiley and Sons, Inc. 192p.

Leung, Y., Mei, C.L. and Zhang, W.X. 2000. Statistical tests for spatial nonstationarity based on the geographically weighted regression model. Envir. and Plan. A 32:9-32.

Liu, J. and Ashton, P.S. 1999. Simulating effects of landscape contexts and timber harvest on tree species diversity. *Ecological Applications* 9:186 -201.

Maltamo, M., Puumalainen, J. and Paivien, R. 1995. Comparison of beta and Weibull functions for modeling basal area diameter distribution in stands of *Pinus sylvestris* and *Picea abies*. Scand. J. For.Res. 10:284–295.

Mateu, J., Uso, J.L. and Montes, F. 1998. The spatial pattern of a forest ecosystem. For. Ecol. Manage. 108:163-174.

Miller, T.E. and Weiner, J. 1989. Local density variation may mimic effects of asymmetric competition on plant size variability. Ecology 70:1188-1191.

Moeur, M. 1993. Characterizing spatial patterns of trees using stem-mapped data. For. Sci. 39:756-775.

Moisen, G.G. and Frescino, T.S. 2002. Comparing five modeling techniques for predicting forest characteristics. Ecol. Model. 157:209-225.

Monserud, R.A. and Sterba, H. 1996. A basla raea increment model for individual trees growing in even- and uneven-aged forest stands in Austria. For. Ecol. Manage. 80:57-80.

Newton, P.F. and Jolliffe, P.A. 1998. Assessing processes of intraspecific competition with spatially heterogeneous black spruce stands. Can. J. For. Res. 28:259-275.

Pàez, A., Uchida, T., Miyamoto, K. 2002. A general framework for estimation and inference of geographically weighted regression models: 1. Location-specific kernel bandwidths and a test for locational heterogeneity. Envir. and Plan. A 34: 733-754.

Penridge, L.K. 1986. LPOINT: a computer program to simulate a continuum of two-dimensional point patterns from a regular, through Poisson, to clumped. CSIRO Institute of Biological Resources, Division of Water and Land Resources, Canberra, Technical Memorandum 86/12. 15p.

Pretzsch, H. 1997. Analysis and modeling of spatial stand structure: Methodological considerations based on mixed beech-larch stands in Lower Saxony. For. Ecol. Manage. 97:237-253.

Pukkala, T. 1989. Prediction of tree diameter and height in a Scots pine stand as a function of the spatial pattern of trees. Silva Fennica 23:83-99.

Ripley, B.D. 1977. Modelling spatial patterns (with discussion). J. Roy. Stat. Soc. B 39: 172-212.

Rouvinen, S. and Kuuluvainen, T. 1997. Structure and asymmetry of tree crowns in relation to local competition in a natural mature Scots pine forest. Can. J. For. Res. 27:890-902.

Shi, H. and Zhang, L. 2003. Local analysis of tree competition and growth. For. Sci. 49: 938-955.

Valentine, H.T., Herman, D.A., Gove, J.H., Hollinger, D.Y. and Solomon, D.S. 2000. Initializing a model stand for process-based projection. Tree Phys. 20:393-398.

Vanclay, J.K. 1994. Modeling forest growth and yield: application to mixed tropical forests. CAB International, Oxon UK. 312p.

Wahba, G., 1990. Spline models for observational data. Philadephia, SIAM.

Weiner, J. 1990. Asymmetric competition in plant populations. Trends Ecol. Evol. 5:360-364.

Wells, M.L. and Getis, A. 1999. The spatial characteristics of stand structure in *Pinus torreyana*. Plant Ecol. 143:153-170.

Wykoff, W.R. 1990. A basal area increment model for individual conifers in the northern Rocky Mountains. For. Sci. 36:1077-1104.

Zhang, L. and Shi, H. 2004. Local modeling of tree growth by geographically weighted regression. For. Sci. 50: 225-244.

Table 1. The initialization of the three example plots.

Spatial pattern	Tree Species	Number of Trees	Minimum DBH (cm)	Maximum DBH (cm)	Plot Size (m^2)	Buffer Zone (m)
Regular	Loblolly Pine	100	7.5	20	10000	10
Random	Loblolly Pine	150	5.0	20	10000	10
Clustered	Loblolly Pine	200	2.5	20	10000	10

Note: DBH is the diameter at the breast height.

Table 2. Descriptive statistics of individual tree measurements before and after the plot projection.

			Regular Plot		
Variable	**N**	**Mean**	**Std Dev**	**Minimum**	**Maximum**
DBH[a] (cm)	100	10.5982	2.7772	5.5300	16.9100
DBH[b] (cm)	95	21.7220	2.5856	14.8200	25.7500
BAG (m^2/tree)	95	0.0090	0.0014	0.0047	0.0106
			Random Plot		
Variable	**N**	**Mean**	**Std Dev**	**Minimum**	**Maximum**
DBH[a] (cm)	150	8.7152	3.2013	2.9400	17.01
DBH[b] (cm)	142	19.2979	3.4833	11.8400	25.8600
BAG (m^2/tree)	142	0.0075	0.0020	0.0033	0.0105
			Clustered Plot		
Variable	**N**	**Mean**	**Std Dev**	**Minimum**	**Maximum**
DBH[a] (cm)	200	7.1711	3.0235	2.5000	16.1600
DBH[b] (cm)	169	17.5802	3.4649	9.6600	25.0200
BAG (m^2/tree)	169	0.0065	0.0019	0.0023	0.0106

Note:

[a] tree measurements before the plot projection.

[b] tree measurements after the plot projection.

BAG is the basal area growth during the 5-year growth period

Table 3. Examples of the impact of the two weighting function on the model estimation with different spatial patterns.

		Spatial Weight			Spatial-attribute Weight												
					DBH			DBH^2			APA			Hegyi's CI			
Plot	Tree	w_{ij} Mean	w_{ij} Std	Predicted BAG	w_{ij} Mean	w_{ij} Std	Predicted BAG	w_{ij} Mean	w_{ij} Std	Predicted BAG	w_{ij} Mean	w_{ij} Std	Predicted BAG	w_{ij} Mean	w_{ij} Std	Predicted BAG	Observed BAG
Regular	A	0.0423	0.1629	0.0099	0.0234	0.1176	0.0099	0.0340	0.1489	0.0100	0.0249	0.1393	0.0099	0.0322	0.1499	0.0099	0.00999
	B	0.0255	0.1163	0.0094	0.0223	0.1128	0.0095	0.0198	0.1105	0.0091	0.0211	0.1105	0.0096	0.0162	0.1057	0.0095	0.00968
	C	0.0311	0.1304	0.0089	0.0368	0.1543	0.0089	0.0181	0.1111	0.0089	0.0256	0.1199	0.0089	0.0199	0.1113	0.0089	0.00886
Random	A	0.0129	0.0894	0.0096	0.0143	0.1052	0.0097	0.0088	0.0843	0.0101	0.0085	0.0845	0.0100	0.0071	0.0839	0.0101	0.01000
	B	0.0115	0.0899	0.0096	0.0094	0.0859	0.0095	0.0086	0.0853	0.0098	0.0091	0.0857	0.0098	0.0072	0.0839	0.0098	0.00950
	C	0.0146	0.0931	0.0077	0.0109	0.0884	0.0077	0.0099	0.0871	0.0076	0.0113	0.0890	0.0076	0.0108	0.0883	0.0076	0.00761
Clustered	A	0.0202	0.1145	0.0061	0.0221	0.1025	0.0062	0.0149	0.1035	0.0062	0.0157	0.1052	0.0064	0.0097	0.0826	0.0064	0.00648
	B	0.0172	0.1047	0.0075	0.0080	0.0799	0.0076	0.0123	0.0926	0.0076	0.0115	0.0865	0.0075	0.0159	0.1077	0.0081	0.00763
	C	0.0179	0.1032	0.0076	0.0118	0.0716	0.0076	0.0123	0.0881	0.0078	0.0129	0.0905	0.0076	0.0073	0.0785	0.0078	0.00787

Table 4. The RMSE of the GWR models with the two weighting functions.

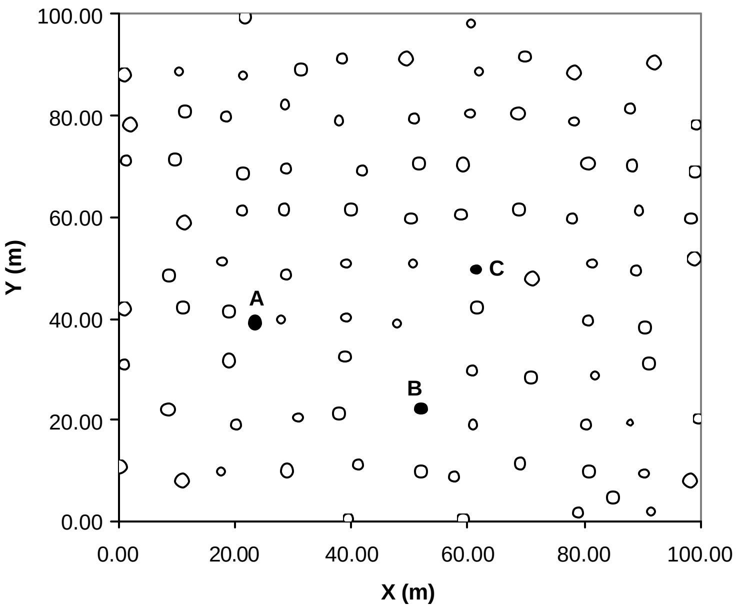
Weighting Function		RMSE		
		Regular Plot	**Random Plot**	**Clustered Plot**
Spatial weighting		0.000068	0.000125	0.000115
Spatial-attribute weighting	DBH	0.000057	0.000105	0.000094
	DBH^2	0.000048	0.000088	0.000114
	APA	0.000055	0.000083	0.000071
	Hegyi's CI	0.000042	0.000079	0.000078

Table 5. The paired t-tests for model residuals and absolute model residuals obtained from the GWR models with different weighting functions.

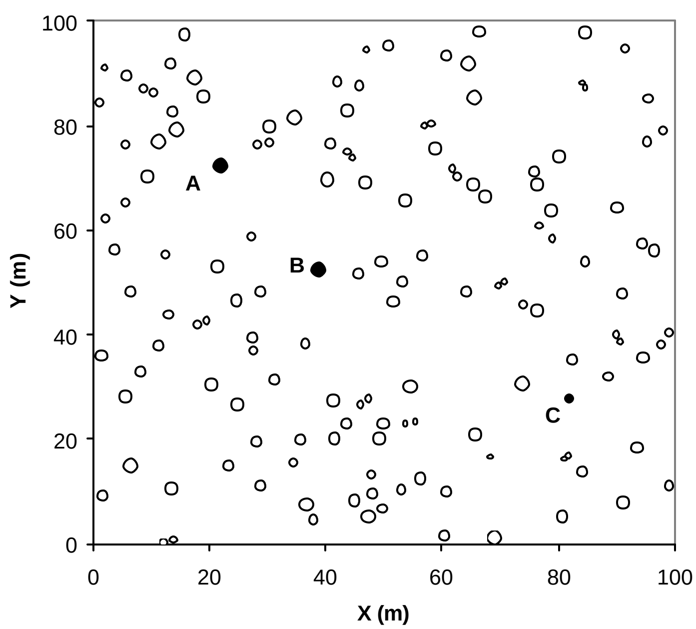
Plot	Attribute	Model Residuals	Model Absolute Residuals
		p-value	p-value
Regular	DBH	0.6850	<0.0001
	DBH^2	0.3019	<0.0001
	APA	0.7868	<0.0001
	Hegyi's CI	0.5036	<0.0001
Random	DBH	0.5243	<0.0001
	DBH^2	0.7046	<0.0001
	APA	0.3295	<0.0001
	Hegyi's CI	0.0638	<0.0001
Clustered	DBH	0.8383	<0.0001
	DBH^2	0.1525	0.0003
	APA	0.1525	<0.0001
	Hegyi's CI	0.0611	<0.0001

Figure 1. Map of tree locations for the three example plots: (a).Regularity; (b). Randomness; and (c). Clustering. (The circle is proportional to the tree DBH).

a.



b.



c.

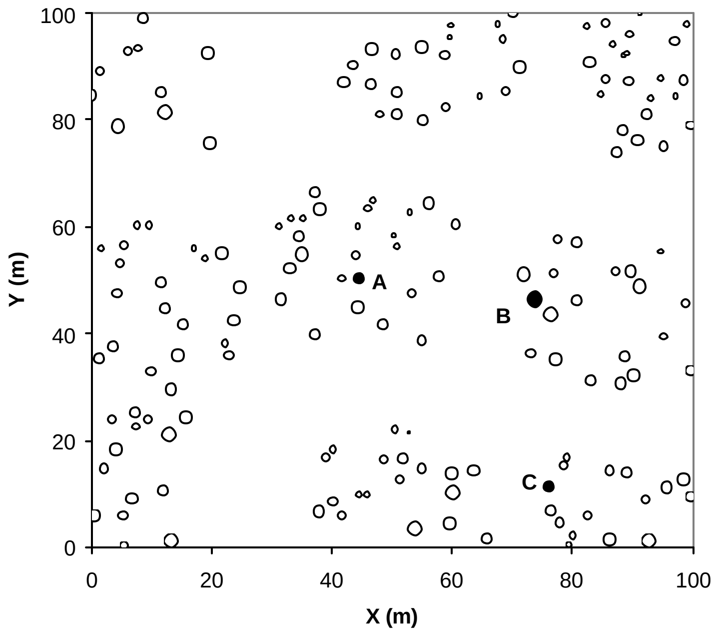
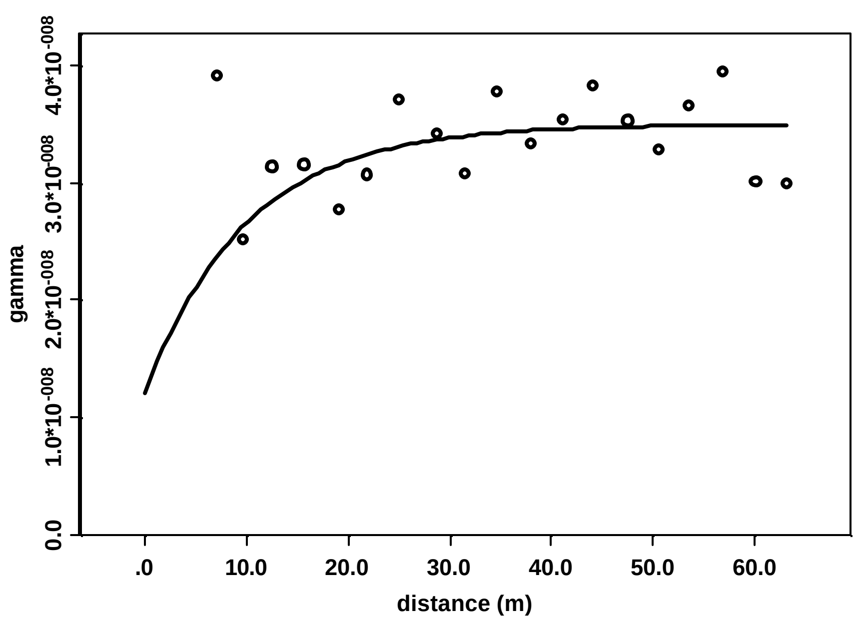
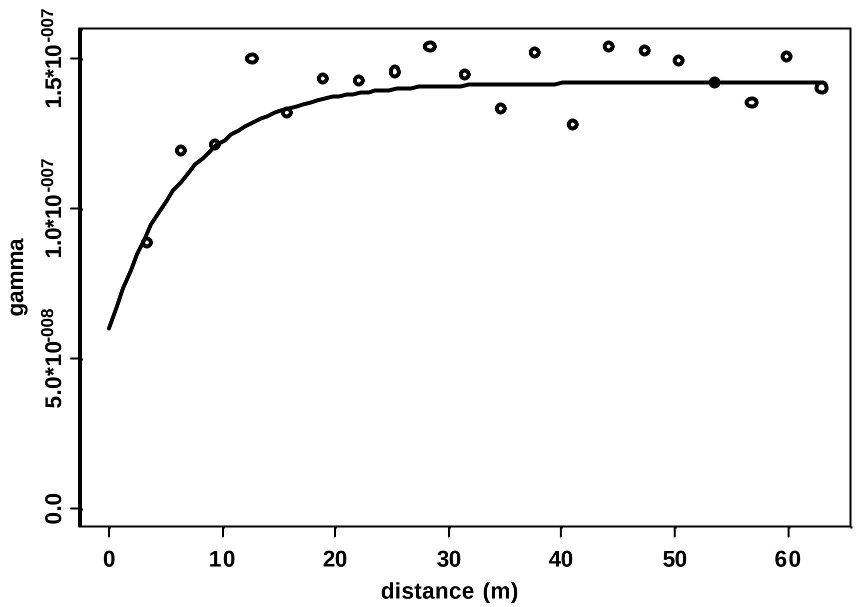


Figure 2. Variogram of the OLS model residuals: (a). Regularity, (b). Randomness, (c). Clustering.

a.



b.


gamma
1.5*10-007
1.0*10-007
5.0*10-008
0.0
0
10
20
30
40
50
60
distance (m)

c.

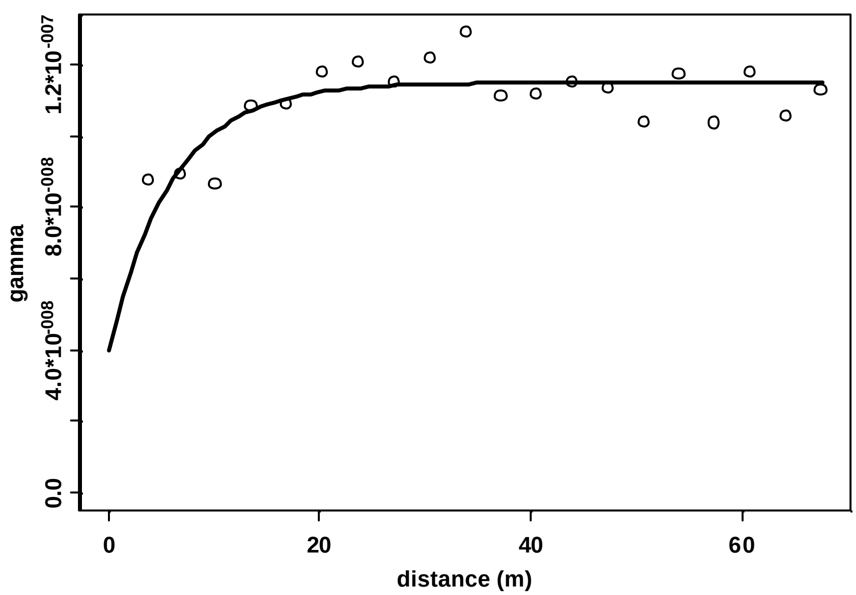
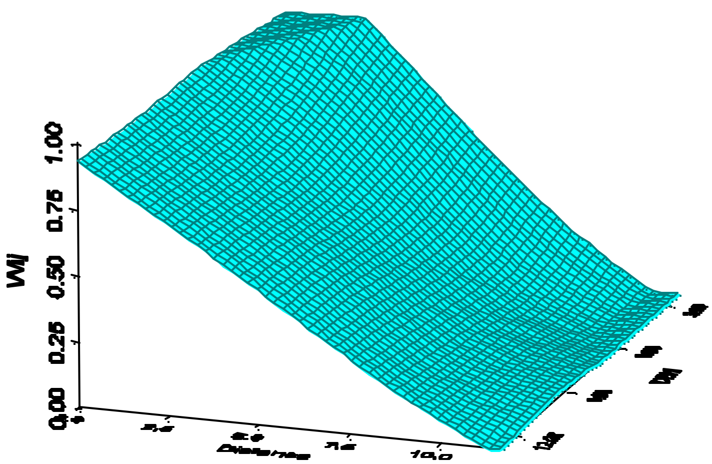

gamma
1.2*10-007
8.0*10-008
4.0*10-008
0.0
0
20
40
60
distance (m)

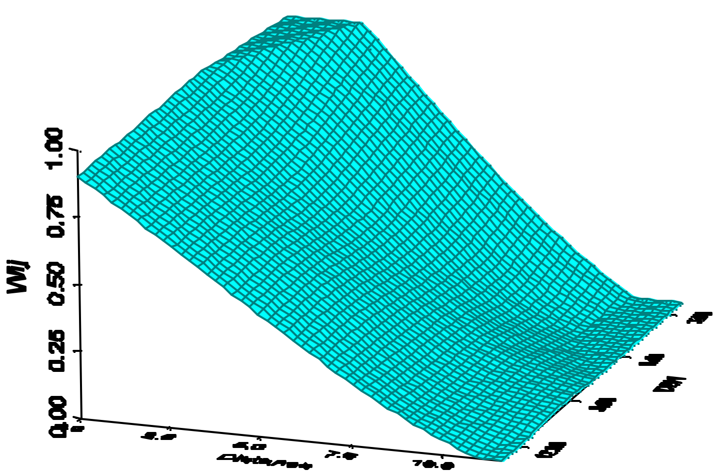
Figure 3. An example of spatial weight and spatial-attribute weight with different tree attributes (i.e., DBH, Hegyi's Ci, APA, and DBH^2): tree A in the clustered plot (Note, for other trees in the regular, random and clustered plots, similar results can be obtained). Distance= the distance (m) between the subject tree and its neighboring trees. Wij=spatial-attribute weight. DBH=diameter at breast height (cm)).

a. Spatial weight; b. Spatial-attribute weight with DBH; c. Spatial-attribute weight with DBH^2; d. Spatial-attribute weight with APA; e. Spatial-attribute weight with Hegyi's CI.

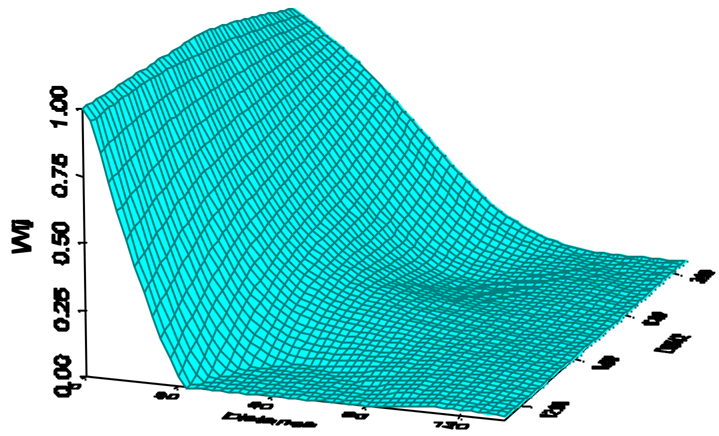
a. Spatial weight



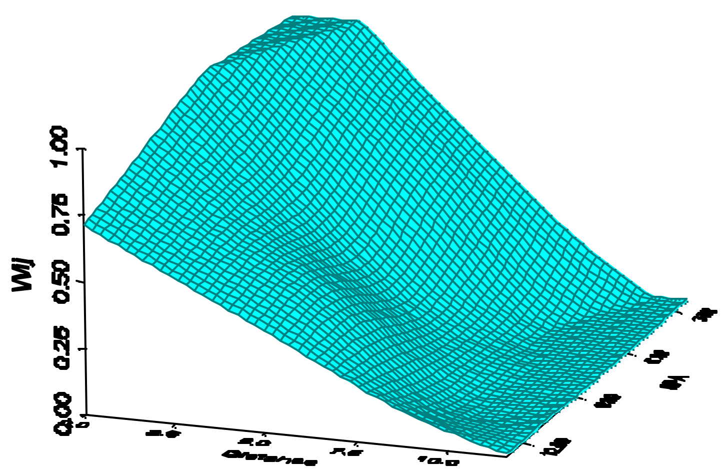
b.



c.



d.



e.

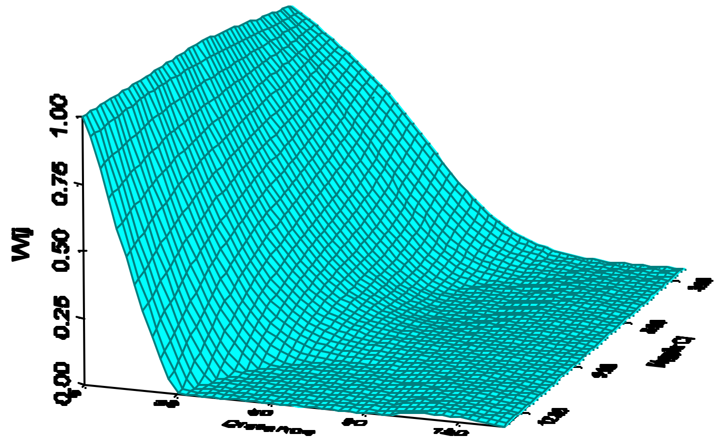
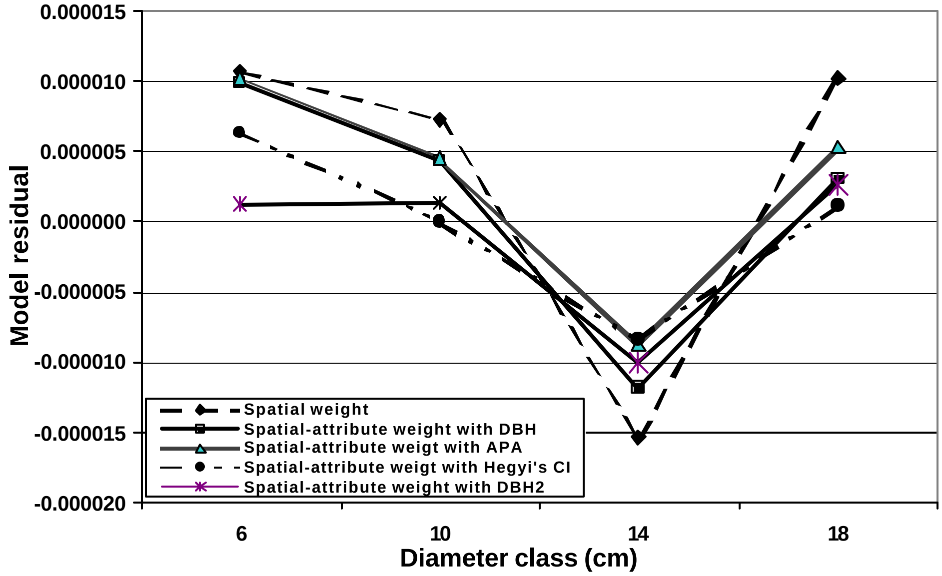
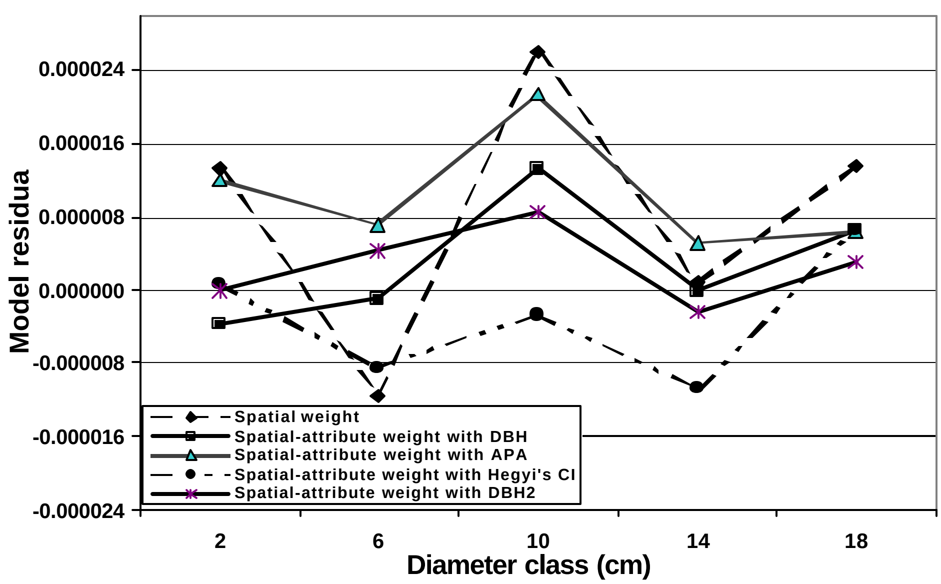


Figure 4. Model residuals across the diameter classes: (a). Regularity; (b). Randomness; and (c). Clustering.

a.



b.



c.

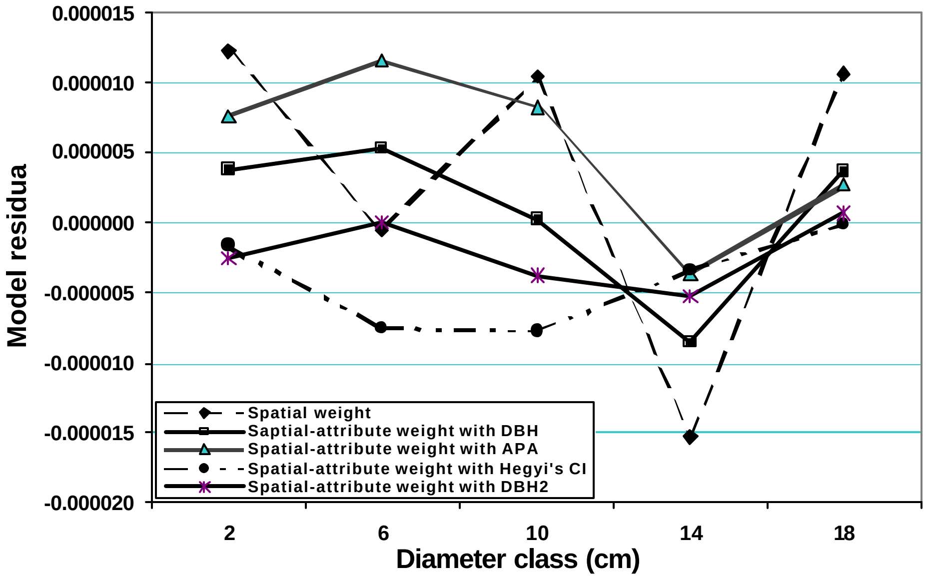
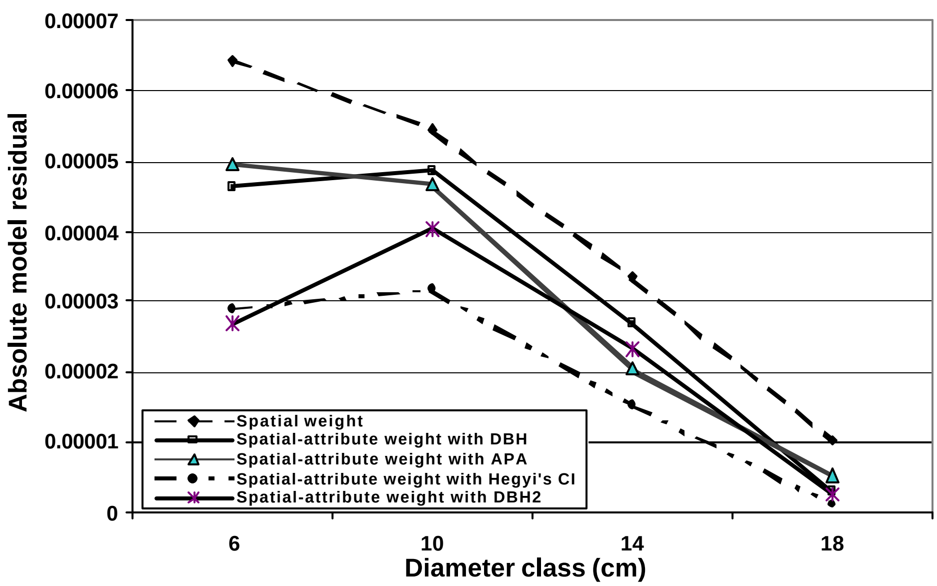
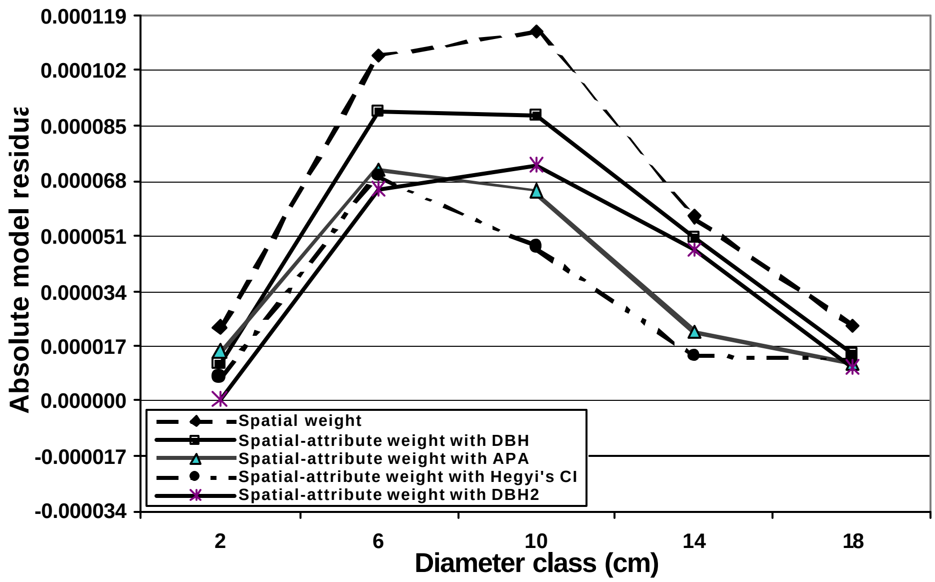


Figure 5. Absolute model residuals across the diameter classes: (a). Regularity; (b). Randomness; and (c). Clustering.

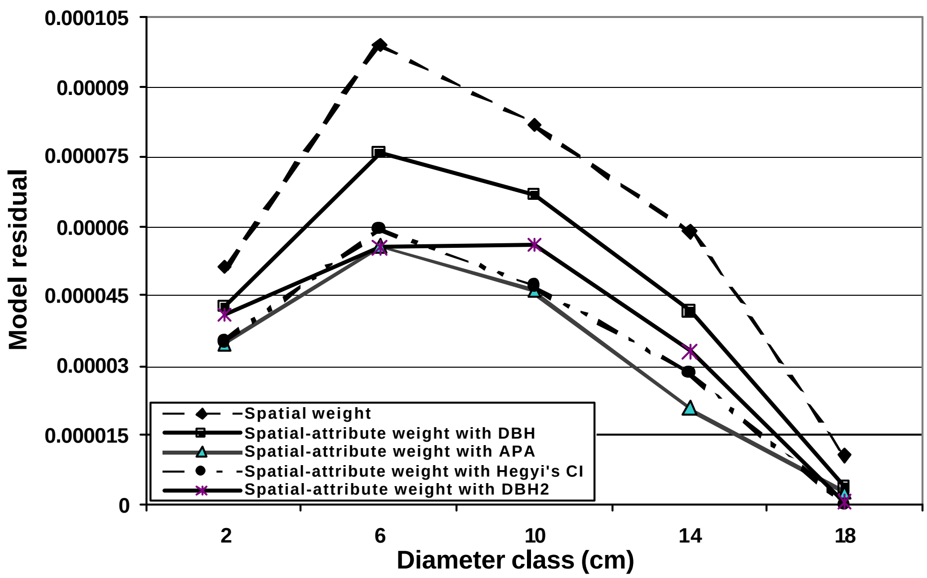
a.



b.


0.000119
0.000102
0.000085
0.000068
0.000051
0.000034
0.000017
0.000000
-0.000017
-0.000034
Absolute model residua
Spatial weight
Spatial-attribute weight with DBH
Spatial-attribute weight with APA
Spatial-attribute weight with Hegyi's CI
Spatial-attribute weight with DBH2
2
6
10
14
18
Diameter class (cm)

c.


0.000105
0.00009
0.000075
0.00006
0.000045
0.00003
0.000015
0
Model residual
Spatial weight
Spatial-attribute weight with DBH
Spatial-attribute weight with APA
Spatial-attribute weight with Hegyi's CI
Spatial-attribute weight with DBH2
2
6
10
14
18
Diameter class (cm)