GeoComputational Approaches to Coverage Maximization in Service Facility Siting

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1. Introduction

Maximizing the capability of sited facilities to provide service to regional demand is a strategic goal of many location planning problems. That is, there are often constraints on the number of facilities that can be sited and given these limitations it is desirable to ensure that regional demand is served to the greatest degree possible. As an example, such goals are often encountered in siting bus stops, emergency warning sirens, surveillance systems, among others.

In many cases, regional demand and potential facility sites can be considered to be continuous, or present anywhere, in a region. For example, tornado warning sirens need to be audible at all locations of human activity. Further, given their relatively small geographic footprint, they can be sited practically anywhere. Typically, facility location problems like those previously mentioned entail making certain assumptions on where demand (e.g., population) is located and where facilities can be sited (Church and ReVelle 1974). While such discretizations of space are beneficial from the standpoint of approaching siting problems using standard optimization techniques, representational issues are known to exist (Drezner and Drezner 1997, Murray et al. 2007).

The focus of this paper is on siting facilities in continuous space to maximally serve continuously distributed demand. Recently, a geometric approach addressing this problem given the siting of a single facility was proposed by Matisziw and Murray (2007). In this paper we focus on the more general situation of siting multiple facilities and propose geocomputational methods for problem solution.

The problem of interest may be specified mathematically as:

$$Maximize \quad \iint\limits_{\bigcup\limits_{j=1,\ldots,p \in G} V_j \bigcap G} \delta(G)\, dA \qquad (1)$$

where,

A= total coverage area (function of selected facilities j)
G = region
$\delta(G)$ = demand function
p = number of facilities to be sited
V_j = service area of facility sited at j.

The objective of this model is to maximize the demand in region *G* that is suitably serviced (or covered). Demand is considered covered if it is within the service area of sited facilities. The number of facilities to be sited is constrained to *p*. This non-linear, non-convex model involves integrating demand density over the area formed by the union of all established service areas within region *G*. Thus, given a continuous distribution of demand across the region, the task is to determine the placement of facilities such that they collectively serve the maximal demand possible.

In what follows, we review literature relevant to this research. Next, we propose a geocomputational approach to solve this problem.

2. Background

The problem of siting facilities in support of serving regional demand can be approached in two basic ways: 1) siting to achieve complete regional coverage and 2) siting to maximize regional coverage. The latter is the focus of this research. Church and ReVelle (1974) were the first to offer a formal mathematical description of this basic problem, known as the Maximal Covering Location Problem (MCLP). The MCLP is a mixed-integer mathematical programming model that seeks to identify the *p* facility sites that maximize demand coverage. In this context, facilities (*j*) have a set service standard *S* reflecting a time/distance range within which demands (*i*) are considered effectively served (e.g. $d_{ij} \leq S$, where d_{ij} is the distance between *i* and *j* and *S* is the maximum allowable service standard). Central to this problem specification is the assumption that demand and possible facility sites can be represented as a set of discrete locations.

Approaches have been proposed to relax spatial assumptions on facility placement in the MCLP to a certain degree. For instance, if demand locations are known, then geometric properties of coverage can be used to identify a set of candidate facility sites in continuous space that contains a subset of *p* sites that maximize coverage (see Church 1984; Murray and Tong 2007). Although facilities are permitted to site in continuous space, discretisation is possible given spatial properties. Such descretisation is possible by assuming either representation of demand as points or uniformly distributed demand in the case of lines or area objects, and enables problem solution using traditional discrete approaches.

What remains a challenge is dealing with continuously distributed demand, particularly in the case of maximal coverage. Related work for complete regional coverage does exist. Specifically, the p-centre problem is one model that can be used to site facilities to ensure complete coverage of a region. The goal of the p-centre problem is to identify the *p* facility sites minimizing the maximum distance from each demand location to its nearest facility. The Voronoi diagram heuristic (VDH) was proposed by Suzuki and Okabe (1995) and Suzuki and Drezner (1996) to solve the continuous space p-centre problem, and subsequently extended in Wei et al. (2006) to address practical application issues. The idea behind the VDH is that one can iteratively find a Voronoi diagram for a given set of p locations, then identify the optimal one centre for each Voronoi polygon. The process continues to iterate as long as a centre location changes.

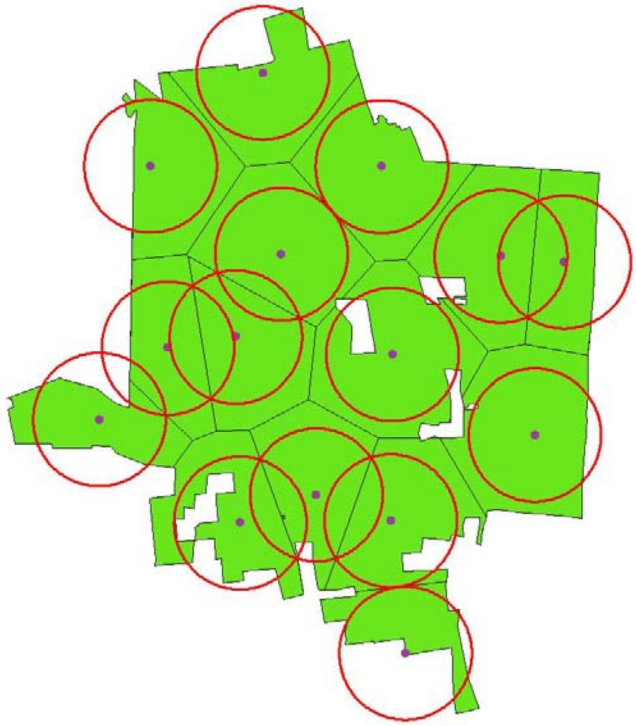

Figure 1. VDH solution for partial coverage.

Unfortunately, the p-centre problem, and hence the VDH, cannot deal with coverage maximization with a limited number of facilities. To illustrate this point, Figure 1 depicts an optimal p-centre solution for *p*=15 using the VDH as well as the effective coverage distance of a warning siren (976 m). For this service standard, some 25 sirens are actually needed for complete coverage. Examining the shown configuration, it is clear that sirens could be shifted to achieve greater coverage for this number of facilities. Given this, the p-centre solution clearly does not optimize coverage maximization.

As noted previously, a special case of the maximal coverage problem was approach by Matisziw and Murray (2007) for a single facility. Their approach to solving this special case relied on exploiting the geometric properties of a region. In particular, an alternative representation of a region, known as the medial axis, is shown to contain an optimal facility site. Thus, finding an optimal location for maximal coverage can be reduced to a search of those locations on the medial axis.

3. Proposed Solution Approach

In this paper, we propose a geocomputational approach for solving the continuous space maximal covering problem using Voronoi diagrams and medial axes. This can be considered an extension of previous work on the *p*-centre problem (e.g., VDH) as well as an extension of the single facility work of Matisziw and Murray (2007).

The proposed approach for solving problem (1) is as follows:

1. Locate *p* facilities.

2. Generate associated Voronoi diagram. Consider all Voronoi polygons unevaluated.
3. For unevaluated Voronoi polygon, less external coverage to area, identify medial axis.
4. Find best facility location along medial axis.
5. If any Voronoi polygon unevaluated, go to step 3.
6. If facility configuration has changed, go to step 2.
7. Stop. Local optima reached.

Computational results obtained from implementing this solution approach will then be assessed in relation to those obtained using discrete methodologies.

4. Conclusions

As discussed in this paper, siting facilities to maximize coverage of regional demand is an important planning goal in a variety of contexts. However, existing approaches entail discretisations of space that can spatially bias modelling results. Here, we relax assumptions of discrete space and seek to maximize coverage of continuous demand through siting facilities in continuous space. Given that this is a non-linear and non-convex spatial optimization problem, geometric techniques in GIScience offer much potential for addressing this complex model.

5. References

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