Computing and calibrating disaggregated spatial interaction models

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1. Introduction

Spatial interaction models for commuting and trip distribution are a core component of transportation modeling in geography and regional science. Because of their long standing development, the fundamental computational and algorithmic issues associated with calibration and estimation of these models are quite well worked out (see papers cited in references and standard texts by Batty (1976); Wilson (1974); and Fotheringham and O'Kelly, 1989). The underlying computational issues are therefore a relatively routine matter of solving systems of non-linear equations with balancing factors. It has become apparent, however, that large scale disaggregate spatial models for trip distribution by employment type, gender, or other key socio-economic variables require at least some careful preplanning to make efficient and useful computations. Apart from good programming practice however, there seems to be some scope for improved mathematical tools for large scale applications. Sensitivity analysis for example can be accomplished through a type of predictor-corrector method. This paper will review and explore such issues in the context of origin-destination trip tables for several US cities. Practical and substantive implications in terms of the excess commuting literature are provided as an empirical test.

2. Project overview

As part of an extensive calibration and sensitivity analysis on US trip distribution models, the authors have preliminary experience with the size of the zoning schemes, and the numerical run times from an initial naïve algorithm (see Figure 1 and Table 1). The original focus of that work was not on computation, so there was no attempt to tune the algorithm to achieve efficient run times. The results show that the extensive run times are a barrier to effective scenario building and have opened the need for faster implementations. Our goal in this paper is to develop improved numerical run times.

The model aims to compute the entropy of the trip distribution as a function of varying trip length, with a view to gauging the degree of difficulty of adapting the trip distribution to shorter, more constrained levels.

City	# of zones	Time (sec.)
Las Vegas	345	50
San Diego	505	39
San Antonio	1059	166
Baltimore	1592	860
Pittsburgh	2161	1573
Denver	2656	3518
Cleveland	2996	5036
Philadelphia	4426	17983

Table 1. Cities in preliminary case study.

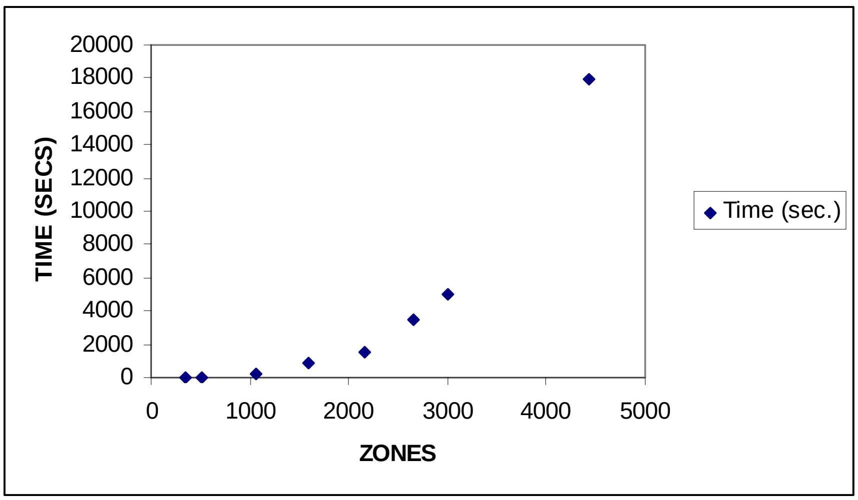

Figure 1. The computational effort increases exponentially with number of zones.

The standard doubly constrained spatial interaction model (Wilson, 1974) is given by:

$$T_{ij} = A_i O_i B_j D_j [f(C_{ij})]$$

where,

A_i = origin balancing factor
O_i = number of worker living in zone i
B_j = destination balancing factor
D_j = number of jobs in zone j
$f(C_{ij})$ = cost function between zones i and j

The cost function, $f(C_{ij})$, is taken to be an exponential function and uses a parameter to model the empirically defined distance decay effect (Fotheringham and O'Kelly, 1989). Thus, the spatial interaction model incorporating the exponential function becomes:

$$T_{ij} = A_i O_i B_j D_j \exp(-\beta C_{ij})$$

The model is especially useful for assessment of the so-called excess commuting phenomenon, as well as in applications that require estimates of unobserved sub-population segments.

Using a standard algorithm to get beta and entropy (based on Fotheringham and O'Kelly, 1989) we input the observed average trip length, and an initial beta value, in this case say 0.1. Then there is a loop for the first stabilization of the balancing factors. If they are stable then the code calculates the predicted trip length; if this is within the permissible error range the code outputs beta; if it is not then it adds an increment to the original beta value and starts the larger loop again and stops when the predicted trip length is within tolerance of the observed trip length.

The exogenous input to the model is the data consisting of the number of workers living in the origin zone (O_i), the number of jobs in the destination zone (D_j), the distance (C_{ij}) and observed flow (T_{ij}) between them. As discussed in O'Kelly and Niedzielski (working paper, 2007) the spatial interaction component provides the necessary calculations, while satisfying these constraints, and incorporating distance-decay effects. The model seeks to fill in the cells of the modified trip distribution maintaining consistency with the observed flows and origins and destinations of the trips while reflecting variations in average trip length.

There are many parameters that influence the computational speed. There is the size of the problem and disaggregation as well as the incremental changes and permissible error range for convergence. When this calibration in embedded in a sensitivity analysis, such as might occur in gauging the impact of decreasing trip length, the starting values for each successive iteration may be derived from the prior step. Such improvements are essential in order to make it possible to perform sensitivity analysis in disaggregated models.

This paper reports improved computational approaches based on better exploitation of mathematical properties of the dependence of the entropy statistic on the trip length (i.e. beta) parameter. The new computational insight in this paper is to exploit the functional relationships between these statistics / parameters and thereby make more efficient computations of the rates of change, and thus the components of the total differential.

The maximum entropy derivation of the doubly constrained trip distribution model is found from:

$$\text{MAX} \qquad H = -\sum_i \sum_j T_{ij} \ln T_{ij}$$

Subject to:

$\Sigma_j T_{ij} = O_i$ λ_i is the related Lagrangean multiplier
$\Sigma_i T_{ij} = D_j$ μ_j is the related Lagrangean multiplier
$\Sigma_i\Sigma_j T_{ij} C_{ij} = C$ β is the is the related Lagrangean multiplier

It can be shown that

$$T_{ij} = \exp (- \lambda_i - \mu_j - \beta c_{ij})$$

which is the first order condition for the objective function to reach a maximum.

As shown in O'Kelly and Niedzielski (2007) we can use these insights to solve the maximum entropy derived doubly constrained spatial interaction model for a given value of the average trip length. For the observed average trip length (C_1), this run results in:

$$H_1 = \Sigma_i \lambda_i{}^*{}_1 O_i + \Sigma_j \mu_j{}^*{}_1 D_j + \beta^*{}_1 C_1$$

where the asterisk is designed to emphasize that these are the solution values and all depend on the exogenous data: O_i, D_j, and C_1. Recall that in the excess commuting literature the O_i and D_j are typically held constant, and the trip length is varied (towards a minimum or maximum).

Since λ_i , μ_j , and β all depend on the change in C, the process is computed numerically: i.e. rerun for a different, *reduced*, average trip length (C_2)

$$H_2 = \Sigma_i \lambda_i{}^*{}_2 O_i + \Sigma_j \mu_j{}^*{}_2 D_j + \beta^*{}_2 C_2$$

Then after a small amount of algebra,

$$H_1 - H_2 = \Sigma_i O_i (\lambda_i{}^*{}_1 - \lambda_i{}^*{}_2) + \Sigma_j D_j (\mu_j{}^*{}_1 - \mu_j{}^*{}_2) + \beta^*{}_1 C_1 - \beta^*{}_2 C_2$$

The difference, $H_1 - H_2$ is the effort or degree of difficulty expressed in terms of the change in entropy, which is precisely the focus of our investigation. It is desired to compute this statistic for large systems, with many zones, and multiple layers of disaggregation, over a systematic evaluation of trip lengths.

We know that as we reduce C_1 to C_2 that $\beta_1 < \beta_2$ and $H_1 > H_2$. While this property is well known, its use in developing sensitivity analysis may be exploited to good advantage, especially where large systems, or highly disaggregated models are in question. Models disaggregated by socio-economic variables add significant complexity to the solution procedure by using exogenous data for various employment categories, gender or race among others. The new model now needs to maintain consistency with observed trips and supply and demand totals for each level of disaggregation as well as the overall aggregate totals.

3. Acknowledgements

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