Novel Structural Analyses of Surface Networks

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1. Surface Networks

Surface Network also called Pfaltz's Graph is a graph-based model of terrains, proposed by Pfaltz (1976). In Surface Network, the nodes are the three most important topographic point features, namely pits, peaks, and passes. The edges of the graph are the ridges and channels. The ridges correspond to the paths between passes to the peaks and the channels correspond to the paths between pits to the passes. Fig. 1 shows a hypothetical island and it's Surface Network. Despite some limitations in the Surface Network model, the natural and intuitive resemblance of the Surface Network model (in comparison to raster DEM or TIN mesh) to real terrains has kept it a topic of active research in various disciplines. Rana (2004) has compiled a collection of the key research articles in Surface Networks.

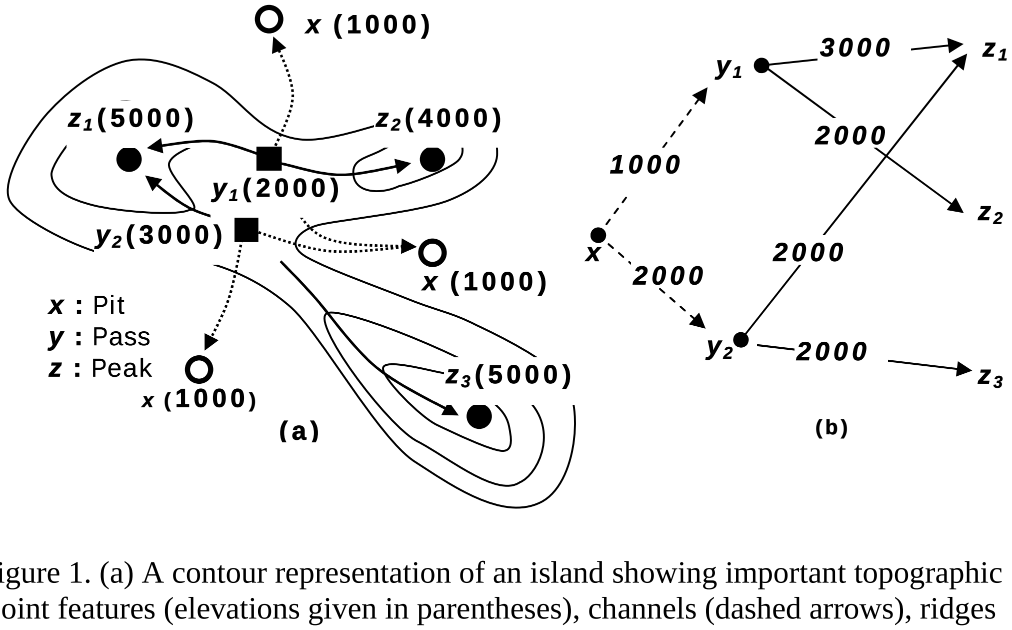

Figure 1. (a) A contour representation of an island showing important topographic point features (elevations given in parentheses), channels (dashed arrows), ridges (solid arrows); (b) corresponding surface network. Numbers on the edges of the graph in (b) are the differences in elevation across the edges.

Surface Networks are amenable for many types of graph-theoretic analyses. Most of the previous researches on structural analysis of surface networks have focussed on their simplification and characterisation. For example, Pfaltz (1976), Mark (1977), Wolf

(1984), Rana (2000), and Edelsbrunner et al. (2003) have proposed various weights and graph contractions (or simplification) that are based on local morphometric (mainly height) properties of the vertices and edges of the graph. However, these types of structural analysis are only a small subset of possible graph-theoretic analyses and measures that could be used to characterise surface networks. In GIS and geography, graphs have been used widely to represent various process/phenomenon such as transport networks (e.g. for routing), drainage networks (e.g. for stream ordering), socio-economic indicators (e.g. for analysis of monetary transactions), and recently cyber-geography (e.g. for trace-route analysis). This paper demonstrates the application of graph measures from non-geomorphological context for surface networks. In particular, this paper presents an experiment to test for Small-world (Milgram 1967) behaviour of Surface Networks and proposes that Surface Networks may not behave as Small-world networks.

The paper also presents novel techniques to refine (i.e. add details) the Surface Network.

Section 1 presents various graph-theoretical measures taken from different disciplines, and their application to characterise the terrain structure in Surface Networks. Section 2 presents novel techniques for the refinement of Surface Networks.

2. Extending the Description of Surface Networks

This section proposes a number of non-terrain related graph measures that are of some relevance in understanding the structure and evolution of terrains as modelled by the Surface Network. Some of these proposed measures include:

- *Length* is the number of edges that are traversed to reach a node from another node. For example, length of a path from a node to all its adjacent nodes is 1.
- *Depth* of a node is the shortest length required to arrive at the node from some other node.
- *Diameter* is the maximum depth in the Surface Network.
- *Mean Depth*, as the name suggests is an average of all the depth of node.
- *Degree* is the number of edges incident on a node.

A Small-world network, as the literal meaning of the term implies, is a random network where relationships between complete strangers can be found using a chain of mutual social acquaintances. Small-world behaviour has been found in a wide variety of networks, ranging from social networks, road maps, food chains, and power grids. Small-world networks have two characteristic properties viz. (i) the degree distribution of nodes exhibit a power-law distribution; (ii) diameter of the network exhibits a power-law when more nodes are added to the network. The networks that have the later property are referred to as scale-free (Albert and Barabási 2002). The primary reason behind these properties is the preference of new nodes to join the network at nodes of higher degree thus forming hubs or clusters of nodes.

Figs. 2 and 3 show the results of graph analyses of the Surface Network of parts of Isle of Man and Latschur Mountains (Austria) respectively, using the software AGNA (Benta 2004). The results indicate characteristic aspects about the terrain for instance,

- The nodes located in the highly incised central parts of both networks have lower mean depth values (Figs. 2a, 3a) thus, depending upon the objective, indicating areas that could be further simplified or areas that ought to be preserved

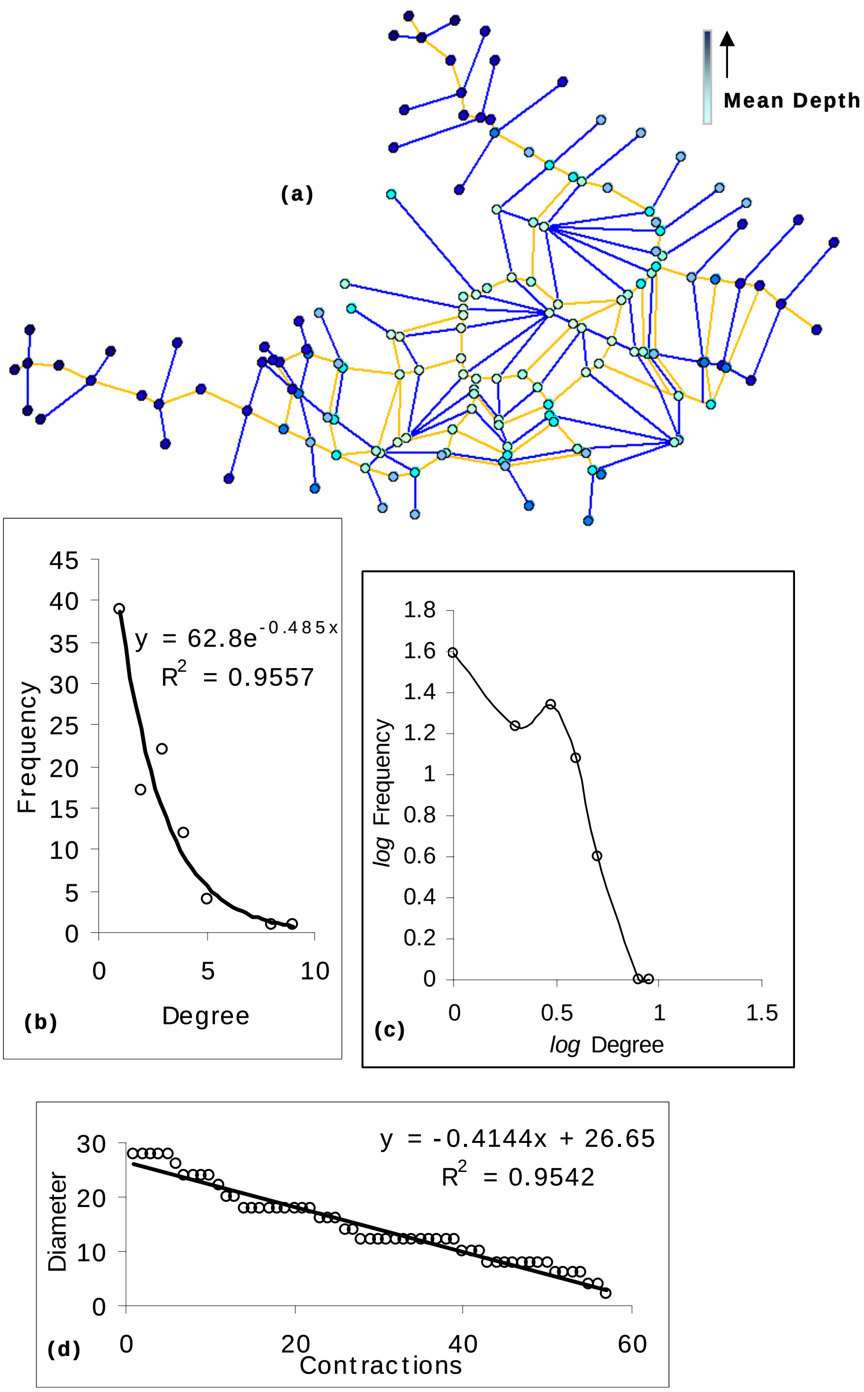

Figure 2. Structural analysis of the Surface Network of a part of Latschur Mountains in Austria. (a) Nodes of the graph shaded according to their Mean Depth; Channels are blue lines and Ridges are orange lines, (b) degree distribution of nodes, (c) degree distribution on a log-log plot, and (d) linear trend of graph diameter with contractions.

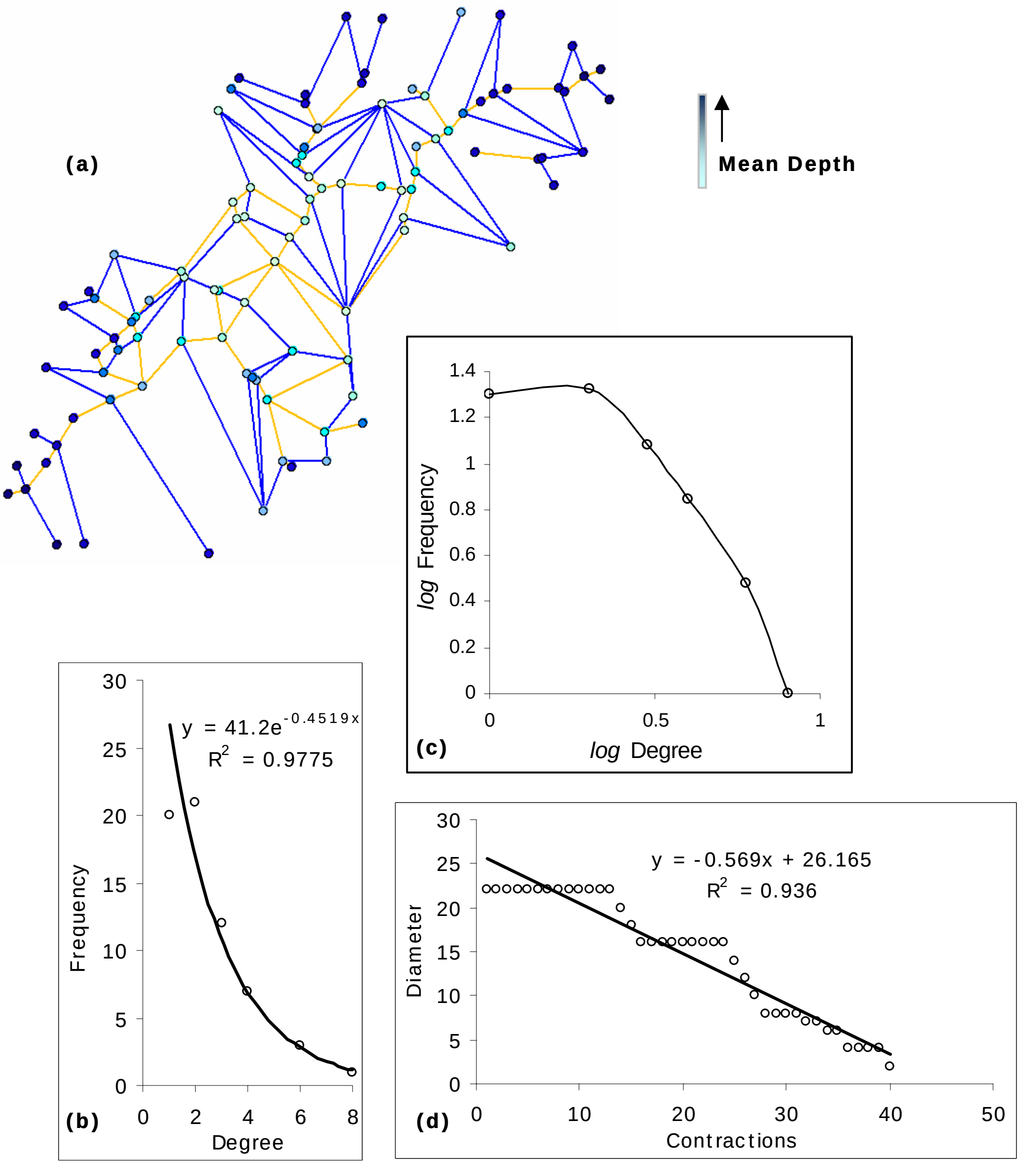

Figure 3. Structural analysis of the Surface Network of a part of Isle of Man. (a) Nodes of the graph shaded according to their Mean Depth; Channels are blue lines and Ridges are orange lines, (b) degree distribution of nodes, (c) degree distribution on a log-log plot, and (d) linear trend of graph diameter with contractions.

- The degree distributions decay exponentially (Figs. 2b, 3b) suggesting that there are clusters e.g. the peak in the northern part of Isle of Man network and pit in the central part of Latschur Mountains network. However, since neither of the degree distributions exhibit a power law on a log-log plot (Figs. 2c, 3c); they are not Small-world networks. Note that the passes were not included in the calculation of degree distribution since all the passes have a degree 4 and thus do not allow randomness essential for Small-world network formation.
- The effect of variation in diameter with addition of nodes was studied under the opposite effect i.e. instead of adding nodes; nodes were suitably removed. The choice of nodes for removal is based on the elevation drop weight measure and the maximum of weighs criterion. The diameters vs. contractions plots (Figs. 2d, 3d) reveal two main insights into the structure of surface networks.
    - There is generally a linear decline in the diameter of the network with the deletion of nodes hence; these Surface Networks are not scale-free networks.
    - The step like pattern of variations suggests that the removal of certain nodes introduce drastic changes in the paths in the network. The flat parts of the plot are contractions that do not affect the overall structure of the network.

3. Refinement of Surface Networks

The term refinement is used for an incremental procedure of adding more details to a data structure. The iterative addition of details is continued until a desired error or the desired level of details has been achieved (Heckbert and Garland 1997). In effect, it is the reverse of the contraction process, which in the case of Surface Networks can be done by either a (y_o, z_o)-contraction or (x_o, y_o)-contraction (Wolf 1984).

The refinement of surface networks promises the following important uses:

- Refinement can be used to simulate landscape evolution processes, for example to study the effect of erosion modelling and simple computer animations.
- Refinement can be used to develop varying levels of details in different parts of the terrain. This could be relevant in the case of incomplete feature extraction and development of multi-scale terrain.

Topological refinement of Surface Networks has not been studied before however Danovaro et al. (2006) have proposed methods to store the multiple Surface Networks that are formed by contractions.

A ridge edge (y_0,z_0) can be split only if the pass y_0 is connected to two distinct pits x_0, and x. With this premise, **(y_0-z_0)-*splitting*** can be defined as follows:

Let,

- W = Surface Network,
- y_o = Pass with Pits R(y_o) = {x_o, x}

Then, after a (y_o, z_o)-splitting W' is the graph with the following properties:

- Vertex set $V(W') = V' = V + \{y', z'\}$,
- Edge set $E(W') = E' = E + \{(y',z_0), (y', z'), (y_0,z'), (x, y'), (x_0,y')\}$
- $h(z')$ is infinitesimally higher than $h(y')$.
- $h(y')$ can be derived by an interpolation of $h(y_0)$ and $h(z_0)$.

Fig. 4 shows the principle of **$(y_0$-$z_0)$*-splitting*** on elementary surface networks. Fig. 5 shows a sequence of refinements of the longest channel of a hypothetical surface network. A rule for splitting a channel edge can be similarly defined.

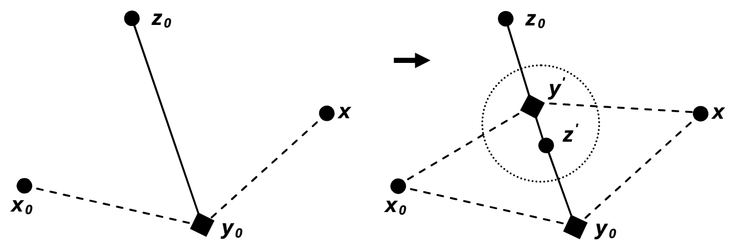

Figure 4. Refining a long ridge with a *$(y_0$-$z_0)$-splitting*. Note how the choice of configuration ensures a topological consistency after the addition of the new edges.

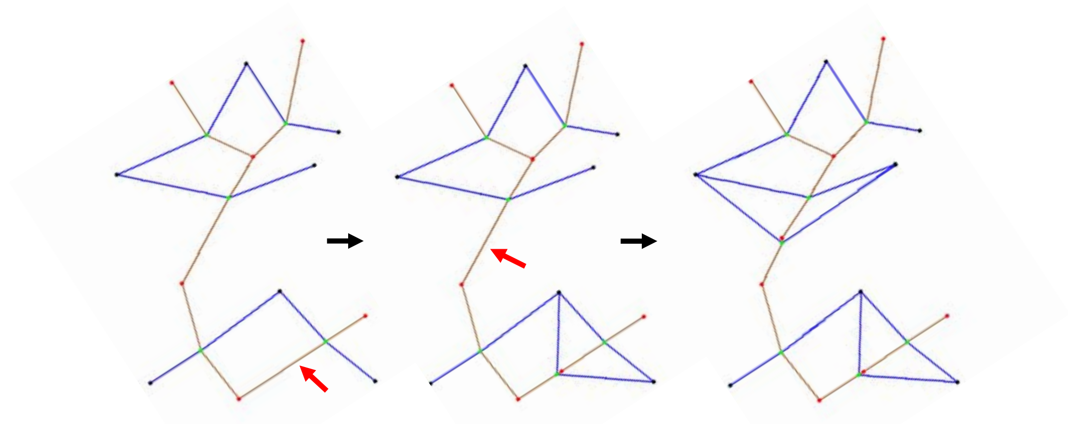

Figure 5. A sequence of 2 refinements on longest ridges (indicated by red arrows) of a Surface Network with repeated *$(y_0$-$z_0)$-splitting*; blue lines- channels, orange lines- ridges, red discs - peaks, green discs – passes, and black discs - pits.

The refined Surface Network is topologically consistent according to the rules proposed by Wolf (1984). The changes introduced in the surface network are reversible i.e. the edges inserted can be removed to restore the surface network to the original state.

5. Summary

Surface Network is a graph-based generic model of terrains. The nodes of the Surface Network are the pits, peaks, and passes. The edges of the graph are ridges (links between passes and a peaks) and channels (links between pits and passes). Due to its resemblance to the natural topography, structural analysis of a Surface Network can provide insights into terrain morphology. The paper demonstrated the use of graph measures such as Mean Depth, Degree distribution and Diameter in order to map the variations in the level of morphological process and whether the Surface Networks exhibit a Small-world behaviour. A topologically consistent approach to add more details i.e. refinement, to Surface Network has been shown that could be useful for simulating terrain evolution and fixing incomplete topology.

6. Acknowledgements

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7. References

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