

# Decentralized computing of topological relationships between heterogeneous regions

Lin-Jie Guan<sup>1</sup>, Matt Duckham<sup>2</sup>

<sup>1</sup>Department of Geomatics, University of Melbourne, Victoria 3010, Australia  
Telephone: (+61) 3 8344 9901  
Email: l.guan4@pgrad.unimelb.edu.au

<sup>2</sup>Department of Geomatics, University of Melbourne, Victoria 3010, Australia  
Telephone: (+61) 3 8344 6935  
Fax: (+61) 3 9347 2916  
Email: matt@duckham.org

## 1. Introduction

A geosensor network (GSN) is a wireless network of tiny, sensor-enabled computing devices, called sensor nodes, which can generate and process fine-grained spatial and temporal information about dynamic environmental phenomena (Stefanidis and Nittel, 2005). In many applications, the topological relationships between spatial regions in the environment are often of special interest. For example, in order to predict bushfires, people would want to know if there is a hot region inside a dry region. This research will investigate computing in a GSN the fundamental topological relationships between spatial regions.

Most prior work on topology computation focuses on processing homogeneous regions (e.g. static topology detection or dynamic tracking of topological changes) (Jiang and Worboys, 2009, Sarkar et al., 2008, Lian et al., 2007). This research aims to compute a set of topological relationships between heterogeneous regions.

The key contribution of this paper is the development of a decentralized approach to computing the topological relationship between spatial regions. Decentralized algorithms rely only on local information about a node's own state and its (one-hop) neighbors' states, with no centralized control. Existing approaches to computing the topological relationships between spatial regions rely on centralized data stores, like GIS or spatial databases, which may be unavailable or inefficient to use in the context of GSN (Estrin et al., 1999).

## 2. Spatial regions in a GSN model

Consider a static GSN densely deployed in a physical environment to monitor some environmental variables such as temperature, humidity or light level. For simplicity of the problem description, it is assumed that the environmental phenomenon of interest is binary. Each sensor node  $v$  has a binary sensor reading  $S_1(v)$  (1 or 0) indicating whether it is inside or outside a spatial region  $R_1$ . The one-hop neighbors of a node  $v$  are denoted as  $nbr(v)$ . It is also initially assumed that the communication network connecting these nodes is structured as a maximally connected planar graph (a triangulation). Later

extensions relax this assumption. Boundary nodes for a spatial region  $R_1$  of interest can be defined as follows:

**Definition 2.1.** A *boundary node* is a sensor node  $v$  in a GSN such that there exists one neighbor  $v'$  in  $nbr(v)$  where  $S_1(v) \neq S_1(v')$ .

A binary value  $B_1(v)$  is used to indicate whether a node is a boundary node of a region  $R_1$ . A natural definition of four sensor node states can then be formulated in terms of their binary sensor reading  $S_1(v)$  and boundary value  $B_1(v)$ :

**Definition 2.2.** *Sensor node state* of a node  $v$  for a region  $R_1$  is defined as:

- Exterior node:  $S_1(v) = 0 \wedge B_1(v) = 0$
- Interior node:  $S_1(v) = 1 \wedge B_1(v) = 0$
- Inner boundary node:  $S_1(v) = 1 \wedge B_1(v) = 1$
- Outer boundary node:  $S_1(v) = 0 \wedge B_1(v) = 1$

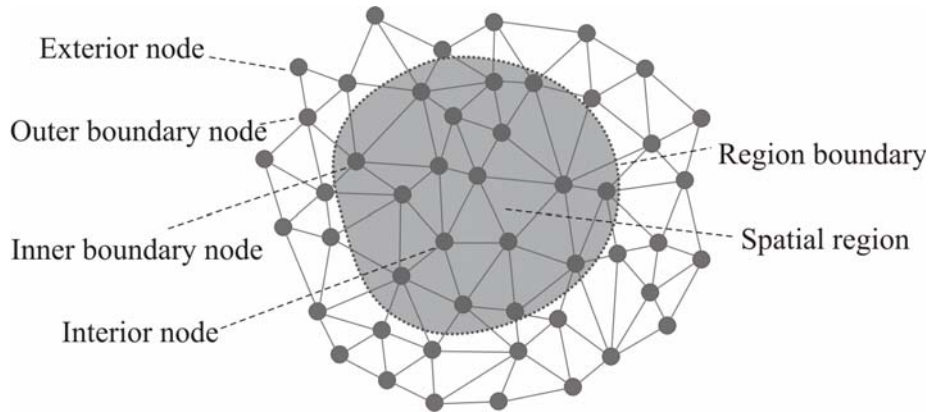


Figure 1. Sensor node states for a spatial region monitored by a GSN.

The four node states of a spatial region  $R_1$  monitored by a GSN are illustrated in fig.1. Notice that the state for each node can be determined in a purely decentralized way, based on local information including nodes' own sensor readings and their one-hop neighbors'.

### 3. Topological relationships between regions in a GSN

A well-known set of fundamental topological relationships between spatial regions have been identified by the four intersection model, comprising disjoint (D), meet (M), contains (C), covers (V), equal (E), overlap (O), inside (I), and coveredBy (B) as shown in fig.2 (Egenhofer and Franzosa, 1991). The computational framework described in this section aims to enable these eight topological relationships between spatial regions to be computed in a *decentralized* manner.

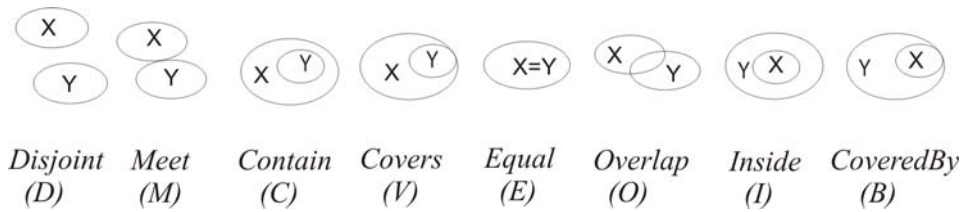


Figure 2. Eight fundamental topological relationships between two spatial regions.

For simplicity, assume two environmental phenomena (e.g. humidity and temperature) are being monitored by a GSN. Each sensor has sensor readings  $S_1(v)$  and  $S_2(v)$  for these two variables. Correspondingly,  $B_1(v)$  and  $B_2(v)$  denote whether it is a boundary node for spatial region  $R_1$  and  $R_2$ . Therefore, the state of a sensor node  $v$  can be represented in the form of a 4-tuple  $(S_1(v), S_2(v), B_1(v), B_2(v))$  which distinguishes  $2^4 = 16$  possible node states in our GSN. Fig.3 shows all 16 node states for two overlapping regions.

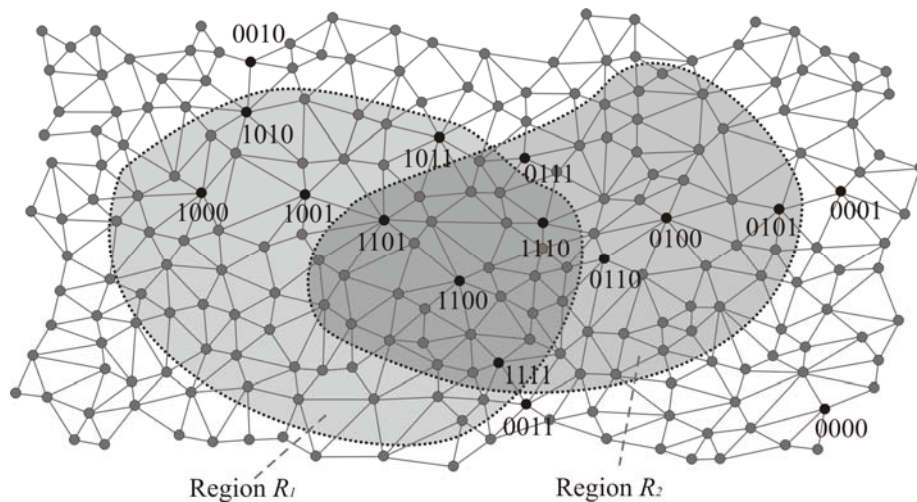


Figure 3. The 16 possible node states in a GSN monitoring two spatial regions.

Depending on the topological relationship between two regions, not all node states are possible. Table 1 shows for each local node state, the set of possible topological relationships that may exist globally.

#	node state	candidate relations
1	1111	O,E,B,V
2	1110	O,B,I
3	1101	O,V,C
4	1010	D,M,O,V,C
5	0101	D,M,O,B,I
6	1100	O,E,B,I,V,C
7	1000	D,M,O,V,C
8	0100	D,M,O,B,I
9	0000	ALL
10	0010	D,M,O,V,C
11	0001	D,M,O,B,I
12	0110	O,B,I
13	1001	O,V,C
14	0111	M,O,B,I
15	1011	M,O,V,C
16	0011	D,M,O,V,B,E

Table 1. Candidate topological relationships determined by 16 node states.

Given a *pair* of node states from one-hop neighboring nodes, a smaller set of topological relationships are expected to be computed locally. Table 2 shows the composition table of possible topological relationships between regions, inferred by two (one-hop) neighboring nodes. The composition table has symmetry about the leading diagonal (as expected since neighborhood is undirected), hence symmetric values are grayed out.

This composition table is largely derived from the intersection of the corresponding sets in table 1, with one main difference. From definition 2.1 and 2.2, an exterior node cannot be adjacent to (one-hop neighboring to) inner boundary node and interior node. Hence, some combinations of neighbors are not possible. For example, the pattern  $(0\bullet0\bullet)^1$  can not be adjacent to the one with pattern  $(1\bullet1\bullet)$  or  $(1\bullet0\bullet)$ . As a result, in Table 2 impossible composites have been eliminated (indicated with the empty set for possible topological relations).

---

<sup>1</sup> The dot symbol ‘•’ indicates that the value can be either 0 or 1.

	1111	1110	1101	1010	0101	1100	1000	0100	0000	0010	0001	0110	1001	0111	1011	0011
1111	OBVE	OB	OV	∅	∅	OBVE	∅	∅	∅	∅	∅	OB	OV	OB	OV	OBVE
1110		OBI	o	∅	∅	OBI	∅	∅	∅	∅	∅	OBI	∅	OBI	∅	∅
1101			OV <sub>C</sub>	∅	∅	OV <sub>C</sub>	∅	∅	∅	∅	∅	∅	OV <sub>C</sub>	∅	OV <sub>C</sub>	∅
1010				DMOV <sub>C</sub>	∅	∅	DMOV <sub>C</sub>	∅	∅	DMOV <sub>C</sub>	∅	∅	OV <sub>C</sub>	∅	MOCV	DMOV
0101					DMOBI	∅	∅	DMOBI	∅	∅	DMOBI	OBI	∅	MOBI	∅	DMOB
1100						OEVCBI	∅	∅	∅	∅	∅	∅	∅	∅	∅	∅
1000							DMOV <sub>C</sub>	∅	∅	∅	∅	∅	OV <sub>C</sub>	∅	MOCV	∅
0100								DMOBI	∅	∅	∅	OBI	∅	MOBI	∅	∅
0000									ALL	DMOV <sub>C</sub>	DMOBI	∅	∅	∅	∅	DMOEBV
0010										DMOV <sub>C</sub>	DMO	∅	∅	∅	MOCV	DMOV
0001											DMOBI	∅	∅	MOBI	∅	DMOB
0110												OBI	∅	OBI	∅	∅
1001													OV <sub>C</sub>	∅	OV <sub>C</sub>	∅
0111														MOBI	MO	MOB
1011															MOCV	MOV
0011																DMOEBV

Table 2. Composition table of candidate node states for (one-hop) neighbouring nodes.

#### 4. Decentralized computing of topological relationships

A key observation from table 2 is that clearly no pair of nodes alone can locally determine the global topological relationship. Instead, we require a computational procedure for combining information between targeted groups of two and even three one

hop neighbors. The full version of this paper sets out such an algorithm. Here, we simply summarize the essential components of this algorithm.

The global topological relationship can be determined using information derived from just six groups (three pairs and three triples) of one hop neighbors. Specifically, the six groupings are:

- A triangle of three one-hop neighbouring nodes with states 1011, 0111, 0011
- A triangle of three one-hop neighbouring nodes with states 1011, 1111, 0011
- A triangle of three one-hop neighbouring nodes with states 0111, 1111, 0011
- A pair of two one-hop neighbouring nodes with states 1111 and 0011
- A pair of two one-hop neighbouring nodes with states 1101, and 10•1 (where • can be either 1 or 0)
- A pair of two one-hop neighbouring nodes with states 1110, and 101• (where • can be either 1 or 0)

The full paper proves that, knowledge of which of these local groupings exist can enable the global topological relationship between two regions to be determined. It further sets out a procedure for efficiently computing in the network the global topological relationship between two spatial regions using targeted communication between these key local groups of one-hop neighbours, aggregating information in the network itself. The full paper also discusses out the relaxation of some of the assumptions, including the requirement for a maximal connected planar graph.

## 5. Conclusion

This paper has presented a theoretical framework for collaboratively computing global topological relationships between spatial regions in a GSN. The decentralized approach exploits spatial correlations between spatial regions and only requires local information including sensor nodes' own information and their one-hop neighbours'. Further work is required on designing efficient decentralized algorithms and testing them empirically in simulations.

## 6. References

- EGENHOFER, M. & FRANZOSA, R. (1991) Point-set topological spatial relations. *International Journal of Geographical Information Systems*, 5, 161-174.
- ESTRIN, D., GOVINDAN, R., HEIDEMANN, J. & KUMAR, S. (1999) Next century challenges: scalable coordination in sensor networks. *Proceedings of the 5th annual ACM/IEEE international conference on Mobile computing and networking*. Seattle, Washington, United States, ACM.
- JIANG, J. & WORBOYS, M. (2009) Event-based topology for dynamic planar areal objects. *International Journal of Geographical Information Science*, 23, 33 - 60.
- LIAN, J., CHEN, L., NAIK, K., LIU, Y. & AGNEW, G. B. (2007) Gradient Boundary Detection for Time Series Snapshot Construction in Sensor Networks. *IEEE Transactions on Parallel and Distributed Systems*, 18, 1462-1475.
- SARKAR, R., ZHU, X., GAO, J., GUIBAS, L. J. & MITCHELL, J. S. B. (2008) Iso-Contour Queries and Gradient Descent with Guaranteed Delivery in Sensor Networks. *The 27th Conference on Computer Communications. IEEE INFOCOM 2008*.
- STEFANIDIS, A. & NITTEL, S. (2005) *GeoSensor Networks*, Boca Raton, FL, CRC Press.