Analysis of Scaling Behavior in Spatial Systems with Large Oscillations of Temporal Fluctuation

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Abstract. Man-made and natural phenomena change over time in highly complex spatial systems. To have a better understanding of and to improve our knowledge about these dynamic phenomena, spatial scientists often rely on time series that manifest the complex dynamics involved. In general, particularly in complex spatial systems, time series are stochastic in nature. That is, time series are usually generated by some non-stationary processes that we need to identify. Of particular interest is the scaling behavior of the time series in the local-global context.

A stochastic process is said to be self-similar if its statistics stay invariant with respect to a change in time scale. Long-range dependent processes are often referred to as fractal processes because their sample path displays self-similarity. That is, the exponent of their moments is restricted by a constant self-similarity parameter. Due to the variations at small intervals, however, non-constant scaling parameter often exists in time series. Nonlinear processes may involve simultaneous appearance of periodic and chaotic behavior. Spatio-temporal intermittency due to parameter fluctuations around some critical values is a common place in non-stationary time series involving multiple scaling behaviors. Stochastic process with multiple scaling is often called an intermittent process. The heavy-tailed distribution is often a power-law type with slow decaying autocorrelation function. Multifractals is a typical multiscaling structure with irregularly varied sample paths. We thus need to discover the multiplicative scheme that generates such multifractal processes.

Conventional multifractal analyses are employed to characterize multifractal properties of normalized and stationary time series. They are generally not suitable for non-stationary time series affected by trends. The existence of trends in time series is actually a common place in many temporal and spatial processes. They might be due to the intrinsic or external conditions generated by some natural or human-made processes. For examples, rainfall and river runoff might have a seasonal periodic trend and population density might have a locational trend. The existence of trends might affect the scaling behavior of processes with long-range dependence. Detrended fluctuation analysis (DFA) is a method for the detection of monofractal scaling properties and the determination of long-range power-law correlation in noisy and non-stationary time series. It attempts to identify different regimes of a system with respect to its different scaling behaviors in the characterizing time series. Crossover time scales are identified according to the change in the correlation properties of the temporal signal. The effect of trends and exogenous/artificial trends having very little relations with the dynamics of the system has also been studied.

To account for multifractality, multifractal detrended fluctuation analysis (MF-DFA), a generalization of DFA, has been formulated to study whether the multifractal nature of the system is solely due to long-range correlation or affected by trends, intrinsic or external, in the time series. Conventional MF-DFA, however, encounters problems when large oscillations exist within the temporal fluctuation. It has been demonstrated that significant errors at crossover locations could be introduced in actual implementation under large oscillations. The present research proposes a method to handle oscillations in the detrending process in order to obtain a smoother approximation of a fluctuation and a more accurate identification of the crossover time scales that separate different regimes. The proposed method is called the temporally-weighted multifractal detrended fluctuation analysis. The idea is parallel to geographically weighted regression which attempts to handle local effects in space by allocating different weights to points with varying distances. Essentially, the effect of points near in space should exert larger influences. In terms of time series, it is generally natural that points near in time are more related than points distance apart. Such principle can then be employed to circumvent the above large oscillation problems in the detrending step of the MF-DFA. The multifractal temporally-weighted detrended analysis is thus constructed in this study to smooth the log-log plot of the fluctuation function, F(s), against the scale, s, so that local effects can be considered and the crossover time scales can be effectively detected. The proposed method is applied to study the scaling behavior of a real-life air temperature series. The proposed moving-window method successfully identifies the crossover time scales, which cannot be detected by the conventional MF-DFA method, separating different temperature regimes. The theoretical arguments are confirmed by the experimental results. The plausible extension of the proposed method to study scaling behavior in the two-dimensional space is also discussed.