1. Introduction

Models of spatial interaction such as transport, migration, commuting and trade usually partition space into zones, to represent the receiving and sending end of the interaction. When zones encompass multiple locations, the partitioning causes an aggregation error (Hillsman and Rhoda 1978). The aggregation error increases with the size of zones. Aggregation errors can cause bias (Goodchild 1979; Openshaw 1984) and when zones are larger than a (generally unknown) threshold, models become invalid (Tobler 1989). It therefore seems obvious to make zones smaller whenever possible. In practice, however, zones often remain large for a number of reasons, including data availability, parsimony and computational complexity.

There are different aspects to the aggregation error; there is the information loss associated with averaging variables and the loss of spatial precision – typically by conceptually concentrating all of a zone in its centroid. Both types of error are amplified when non-linear functions are applied on the aggregated variables, which can lead to a further model bias. One domain where non-linear use of aggregated variables causes a risk of bias is Discrete Choice Modelling where the utility of an alternative is typically an exponential function of descriptive variables. It is therefore well-recognized that aggregation of alternatives must account for the effect of size and variability of those alternatives. However size and variability are often imperfectly understood and the analysis has to depend on judgment, experience and proxy variables (Ben-Akiva and Lerman 1985 p. 252-275). In recent years (micro)simulation has been established as a method for aggregation that circumvents many of the complications of analytical solutions (Train 2009). The location variation however, is not usually considered in simulation applications. For instance Train (2009 p. 55) suggests that alternatives with a geographical dimension require utility parameters specified in a log function to facilitate analytical aggregation. This paper intends to follow the simulation approach and extent it to the issue of geographical aggregation.

2. Method

The model that will be used to test the approach is a doubly-constrained model of commuting. The general doubly constrained model has the following form:

\[ T_{ij} = a_i b_j P_{ij}, \]

where \( T_{ij} \) is interaction between origin zone \( i \) and destination \( j \), in this case the number of commuting trips. \( P_{ij} \) is the prior distribution of interaction from \( i \) to \( j \). \( a_i \) and \( b_j \) are balancing factors, whose values are determined by the constraints respectively at the origin and destination zone. Balancing factors \( a_i \) and \( b_j \) are chosen such that:
\[ R_i = \sum_j T_{ij} \quad \text{and} \quad C_j = \sum_i T_{ij}, \]  

(2)

where \( R_i \) is the constraint for the \( i \)-th row and \( C_j \) is the constraint for the \( j \)-th column, which also implies \( \sum R_i = \sum C_j \). Balancing factors are typically found by iteratively applying the following equations (Fratar 1954):

\[ a_i = \frac{R_i}{\sum b_j P_j}, \quad b_j = \frac{C_j}{\sum a_i P_i}. \]  

(3)

The prior distribution expresses the ‘gravity’ nature of the model, it is defined as follows:

\[ P_{ij} = O_i D_j e^{-\beta d_{ij}}, \]  

(4)

where \( O_i \) is the size of origin zone \( i \) and \( D_j \) is the size of destination zone \( j \). In the case of commuting, origin size is the working residents and destination size is the number of workplaces. \( d_{ij} \) is the distance between zones \( i \) and \( j \) and parameter \( \beta \) the sensitivity to distance.

The doubly constrained model is linear except for the exponential function of distance. The simulation approach will therefore focus on that function. In the traditional approach the prior distribution is calculated on the basis of mean distance between zones:

\[ P_{ij}^{\text{traditional}} = O_i D_j e^{-\beta \bar{d}_{ij}}, \]  

(5)

where mean distance is the distance between zone centroids, with the intrazonal distance being approximated by the ‘internal radius’:

\[ \bar{d}_{ij} = \begin{cases} \left\| c_i - c_j \right\| & \text{if } i \neq j \\ \sqrt{A_i / \pi} & \text{if } i = j \end{cases}, \]  

(6)

where \( c_i \) is the centroid of zone \( i \) and \( A_i \) is some measure of the land area of zone \( i \).

This paper proposes the following alternative:

\[ P_{ij} = O_i D_j \frac{1}{n} \sum_{n=1}^{\delta} e^{-\beta d_{ijn}}, \]  

(7)

where \( d_{ijn} \) is the \( n \)-th random sample of distance between locations in zones \( i \) and \( j \):

\[ d_{ijn} = \left\| p_{im} - p_{jn} \right\|, \]  

(8)

where \( p_{im} \) and \( p_{jn} \) are random locations within respectively zones \( i \) and \( j \). The random locations are drawn from a uniform spatial distribution: a random location in a zone is found by a series of geometrical operations on the polygon that outlines the zones; First the polygon is decomposed into triangles using a dedicated triangulation library (Shewchuk 1996); Next one triangle is randomly selected using the area of each triangle as the weight; Finally a point is found within the selected triangle by applying the algorithm of Turk (1990).

3. Case study and results

The model is applied on commuting data of England as measured by the U.K. Census of 2001 at the level of Standard Table Wards (‘wards’ from here) as well as Local Authority Districts (‘districts’ from here). The data used is available from Centre for Interaction Data Estimation and Research (http://cider.census.ac.uk). Wards form the most detailed geography at which Census commuting data is made available. Districts present a more aggregated geographical level at which practical policy analysis is often carried out. There are 354 districts and 7932 wards in England. The digital boundaries (as polygons) of districts and wards come from UK Borders.
The centroids of zones are calculated as their geometric centre. Fig. 1 presents the ward and district geographies.

The model has been calibrated twice, with both versions of priors (i.e. equations 5 and 7). A bracketing approach called Golden Section Search (Press 1992) was followed to find the value of $\beta$ that minimized the following error:

$$\delta = \sum_{i,j} \left( T_{ij}^{model} - T_{ij}^{census} \right)^2,$$

where $\delta$ is the discrepancy between modelled and actual (Census) commuting matrices.

Table 1 gives estimated values for $\beta$ and the associated error $\delta$. It shows that for the case of wards it makes little difference which approach is chosen, but for districts there is a marked difference in performance where the simulation based model performs 35% better than the traditional model. The graphs in fig. 2 depict the trip distribution as a function of distance and confirm the difference in performance.

Table 1. Calibration results and errors. Note that errors are only comparable between models applied at a common geography.

<table>
<thead>
<tr>
<th>Geography</th>
<th>Model</th>
<th>$\beta$</th>
<th>$\delta(10^9)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wards</td>
<td>Traditional</td>
<td>0.34</td>
<td>2.99</td>
</tr>
<tr>
<td>Wards</td>
<td>Simulation</td>
<td>0.36</td>
<td>2.87</td>
</tr>
<tr>
<td>Districts</td>
<td>Traditional</td>
<td>0.37</td>
<td>90</td>
</tr>
<tr>
<td>Districts</td>
<td>Simulation</td>
<td>0.31</td>
<td>58</td>
</tr>
</tbody>
</table>

Figure 1. Study area England at district (left) and ward (right) levels of aggregation.

Figure 2. Census and modelled trip distributions. Note zone sizes distort distribution patterns particularly at the district level.
4. Conclusion
This paper follows up on the recommendation of Train (2009) and others to employ simulation when faced with discrete choice models for which analytical models are not feasible or too restrictive. The case study is carried out on the generic doubly-constrained model, which is readily generalisable to more sophisticated random utility models.

By comparing two cases that differ in the level of spatial aggregation it became clear that location sampling does significantly reduce the error caused by using average distances. At the fine scale of wards the effect of error reduction is small although still apparent. At the coarser scale of districts however, simulation would seem essential in future models to contain the aggregation error.

Simulation can be a mechanism for reliable modelling on the basis of coarse scale data when fine scale data is not available. An example of such data is the UK Census commuting data that only offers thematically refined data at coarse spatial scales, for instance commuting patterns specified by industry and socio-economic group which allow segmented modelling of commuter behaviour.

5. Acknowledgments
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6. References