# Road Network Analysis using Geometric Graphs of *β*-skeleton

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## 1. Introduction

There exists a numerical analysis of a road network from various viewpoints: the morphological proximity of road networks to typical geometric graphs (Tanimura and Furuyama 2002, Watanabe 2005), the efficiency of travel in a road network (Koshizuka and Kobayashi 1983), the street hierarchies from the multiple perspectives of topology and geometry (Jiang 2009) and so on. In the present study, we employ geometric graphs based on $\beta$-skeleton, which change in response to variations in parameter values, and attempt to analyze road networks by considering the morphological proximity (topological perspective) and the efficiency of travel (geometric perspective).

## 2. Road Network Analysis from Topological Perspective

### 2.1 Concept of *β*-skeleton

Given a spatial distribution of points $p_i$ ($i$ = 1, 2, …, $n$) in two-dimensional space, let us consider various ways of creating geometric graphs that connect the points to each other. As shown in fig. 1, let us assume that two circular arcs pass through the arbitrary points $p_1$ and $p_2$. The size of the closed region $E$ enclosed by the arcs (the crosshatched portions in fig. 1) varies with the parameter $\beta$ ($\geq 0$), such that the area of $E$ increases as $\beta$ increases. Then, if some third point is included within $E$, then the segment with endpoints $p_1$ and $p_2$ is not an edge in the graph, whereas if no such third point is included, the graph contains this segment as an edge.

A geometric graph created according to this rule is called the $\beta$-skeleton (Wang et al. 2003, Bose et al. 2009). It is well established that the case $\beta$ = 0 corresponds to the Delaunay triangulation of the set of points, $\beta$ = 1 corresponds to the Gabriel graph, and $\beta$ = 2 corresponds to the relative neighbourhood graph.

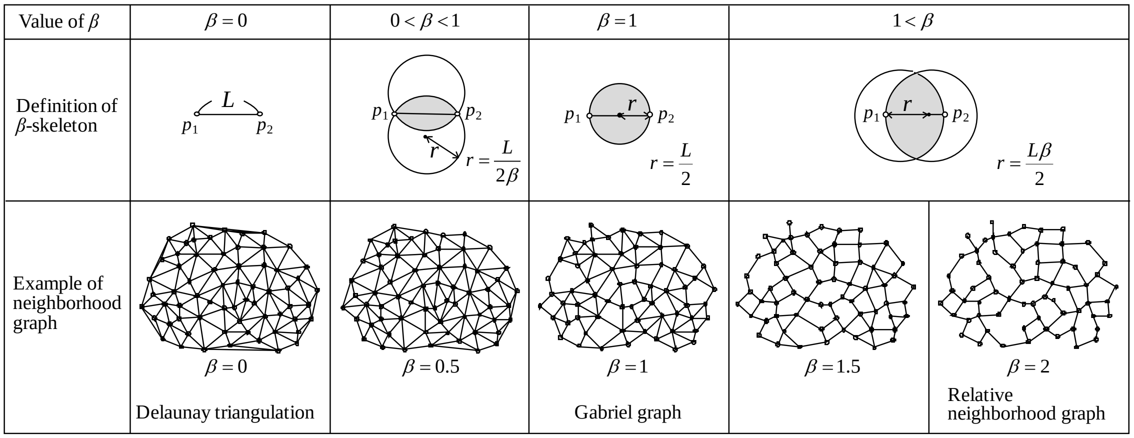


Figure 1. Definition of $\beta$-skeleton.

## 2.2 Definition of agreement rate

Let us define an "agreement rate" as an index expressing how closely the morphology of an actual road network resembles that of a geometric graph. The set of edges making up the road network is denoted by $R$ and that of the geometric graph is denoted by $G$. The number of elements in the set of edges is written as the function n( ). Then, we define the agreement rate ($C$-ratio) as the number of elements in $R \cap G$ divided by the number of elements in $R \cup G$, that is, $n(R \cap G)/n(R \cup G)$.

## 2.3 Maximum agreement rate and value of $\beta$

The greater Tokyo metropolitan region was chosen for the study area, and subdivided into eight sub-regions shown in fig. 2.

Geometric graphs were created for various values of $\beta$, and the resulting agreement ratios with respect to the actual road network were calculated (fig. 3). The value of $\beta$ yielding the maximum agreement rate is labelled $\beta_1$. Table 1 shows the maximum agreement rate and the corresponding $\beta_1$. As shown, the values of $\beta_1$ for the sub-regions lie between 1.0 and 1.5.

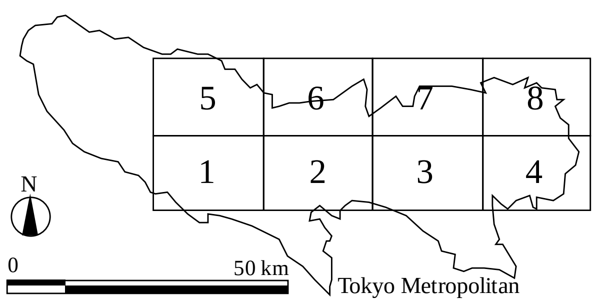


Figure 2. Study area.

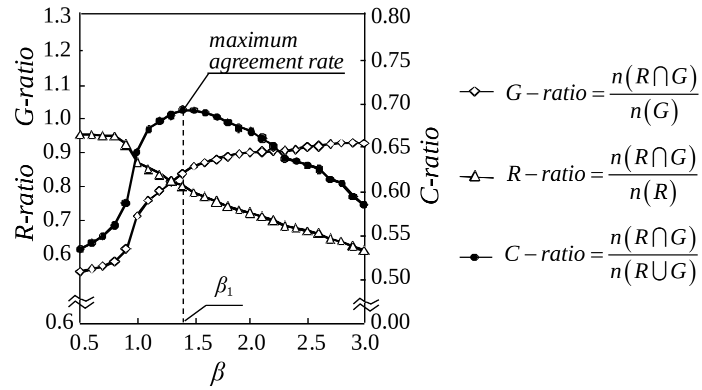


Figure 3. Agreement rate as function of $\beta$ (sub-region 4).

| Sub-region | Maximum agreement rate | $\beta_1$ |
|---|---|---|
| 1 | 0.610 | 1.40 |
| 2 | 0.643 | 1.45 |
| 3 | 0.639 | 1.15 |
| 4 | 0.693 | 1.40 |
| 5 | 0.623 | 1.20 |
| 6 | 0.614 | 1.20 |
| 7 | 0.637 | 1.30 |
| 8 | 0.656 | 1.25 |

Table 1. Maximum agreement rate and the corresponding value of $\beta_1$.

## 3. Road Network Analysis from Geometric Perspective

### 3.1 Concept of spanning ratio

The spanning ratio (*SR*) has been suggested as an index expressing the travel efficiency through a network (Wang et al., 2003). *SR* is defined as the value of the distance $L$ between two points on the network paths divided by the Euclidian distance $D$ between the points. In other words, the greater the values *SR*, the lower the travel efficiency in the network.

### 3.2 Spanning ratio of road network and geometric graph

The intersection points in the road networks $R$ in the previous section were used to create Geometric graphs for various values of $\beta$ ($1.0 \leq \beta \leq 2.0$). Next, two intersections at a time were extracted at random and the value of *SR* was calculated for that pair. The mean $m$ and standard deviation $\sigma$ were calculated for the *SR* of 1,000 point pairs for each graph. The results showed that $m$ is an increasing linear function of $\beta$ ($m = a\beta + b$; $a$ and $b$ are unknown parameters). The increase in $m$ is due to Geometric graphs with higher values of $\beta$ having lower numbers of edges, decreasing the efficiency of spatial motion in the graphs.

Also, the results showed that the value of $\sigma$ grows with the value of $\beta$. The growth of $\sigma$ indicates that there is high variation in the travel efficiency between point pairs, that is, that there is a large difference between the Euclidian distance and the distance in the network between point pairs. Therefore, it is preferable to conduct analysis of spatial motion in regions with low road densities on the basis of distance in the network rather than on the basis of Euclidian distance.

The mean $m$ of *SR* for 1,000 point pairs was calculated for the actual road network of each sub-region. The values of $\beta$ ($\beta_2$) were then inversely estimated using $m$ by the equations ($\beta_2 = (m - b)/a$). Specifically, the values of $\beta$ for the geometric graph indicating the mean values of *SR* equivalent to that of the actual road network were calculated. These values are shown in table 2 along with the corresponding values for parameters of regression equations. As shown, in all the sub-regions analyzed here, $\beta_2$ remains within the range 1.0 to 1.5, the same as $\beta_1$.

| Sub-region | $m$ | $a$ | $b$ | $R^2$ | $\beta_2$ |
|---|---|---|---|---|---|
| 1 | 1.224 | 0.217 | 0.913 | 0.993 | 1.440 |
| 2 | 1.196 | 0.184 | 0.934 | 0.993 | 1.432 |
| 3 | 1.155 | 0.184 | 0.946 | 0.981 | 1.146 |
| 4 | 1.166 | 0.145 | 0.968 | 0.993 | 1.363 |
| 5 | 1.184 | 0.213 | 0.906 | 0.998 | 1.310 |
| 6 | 1.194 | 0.238 | 0.874 | 0.994 | 1.350 |
| 7 | 1.178 | 0.202 | 0.914 | 0.989 | 1.310 |
| 8 | 1.210 | 0.207 | 0.918 | 0.995 | 1.374 |

Table 2. Value of $\beta_2$ for the geometric graph whose travel efficiency is equivalent to that of road network.

### 3.3 Relation between $\beta_1$ and $\beta_2$

Figure 4 shows relationships between the $\beta_1$ (value of $\beta$ for morphological proximity) and the $\beta_2$ (value of $\beta$ for similar travel efficiencies). In sub-regions 1, 5, and 6, suburban areas with low densities of roads, $\beta_1 < \beta_2$ holds. In these areas, there is a risk that using Geometric graphs (the geometric graph for $\beta_1$), which have been created on the basis of morphological proximity, will provide erroneous predictions of travel efficiency. Specifically, the travel efficiency in the actual road network is likely to be lower than that in the geometric graph created on the basis of morphological proximity. On the other hand, $\beta_1$ and $\beta_2$ are roughly similar in sub-regions 2, 3, 4, and 7, the downtown Tokyo area, where the density of roads is high.

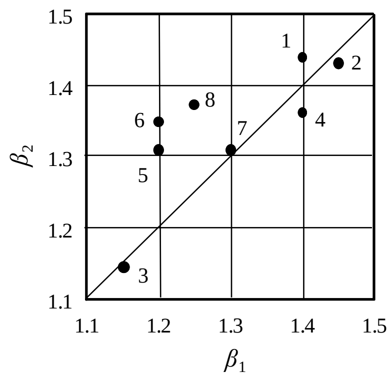


Figure 4. $\beta_1$ versus $\beta_2$ (numerals are sub-region numbers).

## 4. Summary and Conclusions

We carried out an analysis of a road network from each of two viewpoints, network morphology and travel efficiency, by using the concept of $\beta$-skeletons. The following findings were identified:

(1) The value of $\beta$ in a geometric graph with a maximal morphological proximity to an actual road network is in the range 1.0 to 1.5 for the networks examined here.

(2) The agreement rate between a road network and a geometric graph is less in mountainous suburban areas or similar areas with low densities of roads.

(3) The travel efficiency (*SR*) between two points shows more variation in suburban areas with low densities of roads; therefore, when investigating the travel efficiency between locations, the analysis must employ the distance in the network rather than the Euclidian distance between the points.

(4) The value of $\beta$ when there is high morphological proximity between a road network and a geometric graph ($\beta_1$) was nearly equal to the value of $\beta$ when there is a strong similarity between the travel efficiencies in the actual network and the graph ($\beta_2$) in the central part of Tokyo. However, $\beta_1 < \beta_2$ in the Tokyo suburbs, indicating that an analyst must take account of the higher travel efficiency in the geometric graph mostly strongly resembling the actual road network than that in the actual road network itself.

In this paper, we compared the properties of geometric graphs to real road networks. This approach can be extended for the general modelling of various numerical simulations, as well as theoretical analysis on intersections which are randomly distributed following the Poisson distribution.

## 5. References

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