Vector-based Mathematical Morphology

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1. Introduction

Mathematical Morphology (MM) is a mature theory and technique originally developed for processing raster-based binary images. Over time MM was extended to include grayscale functions and images. Today, the generalization of MM to complete lattices is widely accepted as MM's theoretical foundation.

Dilation and erosion are the basic operations of MM; dilation expands a figure uniformly, while erosion shrinks a figure uniformly. Expanding and shrinking can be altered in different directions by employing a non-circular structure element such as a diamond shape to constrain MM operations. A sequential combination of dilation and erosion operations can generate various outcomes from a unique figure. It is very useful in some cases to transform the shape of a figure or extract graphic information from figures. For example, a dilation operation plus an erosion operation may remove small holes and the same combination of operations also can determine if two figures are separate or detached. Analogous combinations of operations are widely used in image processing.

Such transformation and extraction of potential information from figures is also required in vector-based datasets and applications. MM, however, cannot be directly applied to vector data. Thus, vector data must be transformed into raster data before MM processing operations can be executed. After these operations, the vector to raster transformation must be reversed if vector is the final data type required. Unfortunately, precision and information are lost during the two transformations.

In the past, vector-based data processing was considered as computationally intensive. Therefore, raster data were used as an intermediate format to implement some complex algorithms. Nowadays, enhanced computing power supports efficient vector data processing capabilities. Now it is possible to implement the same figure transformation and information extraction from vector data as described above by developing vector-based MM (VMM). Vector-based MM (VMM) is the focus of this paper. Vector-based dilation and erosion operations are defined so as to directly transform vector figures and extract potential information from vector data. The primary eexperiment proves that such definitions are of some interesting features and may potentially grow to a systematic methodology.

2. Vector-based operations and structure elements

2.1 Dilation and Erosion

The two basic operations of VMM, dilation ⊕ and erosion Θ, are defined as outward buffer and inward buffer of a vector figure. Figure 1 gives an example of dilation operation.

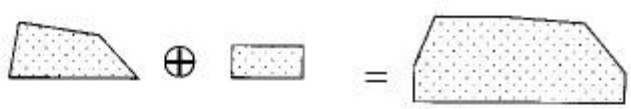


Figure 1. An example of dilation operation (Shi and Wu, 2003)

The buffer operation is now an ordinary operation in commercial GIS software. However, as a fundamental VMM operation, the basic algorithm must be more efficient; to implement some actions constraints may be set for a buffer operation.

Due to the differences between raster and vector data sets, VMM displays some new dilation and erosion operation features. For example, in VMM, $(A \oplus s) \oplus s = A \oplus (s \oplus s)$ and $(A \Theta s)\ \Theta s = A\ \Theta (s \oplus s)$ holds true, even if s is a circle. In MM, errors may occur since the dilation and erosion operations are based on the pixel unit, and not continuous in the 2D plane.

2.2 Structure Element

A structure element is an atomic figure used to generate various different shapes from a single figure. It is at the core of operations in raster-based MM. Likewise, for the vector-based operations defined above in section 2.1, outward and inward buffers can be considered as circle structure elements. We also can use non-circle structure elements. For example, a group of points can generate a buffer with an eclipse as a structure element to indicate the point-source pollution area. Figure 2 shows two examples of different results of a shape dilated with different structure elements.

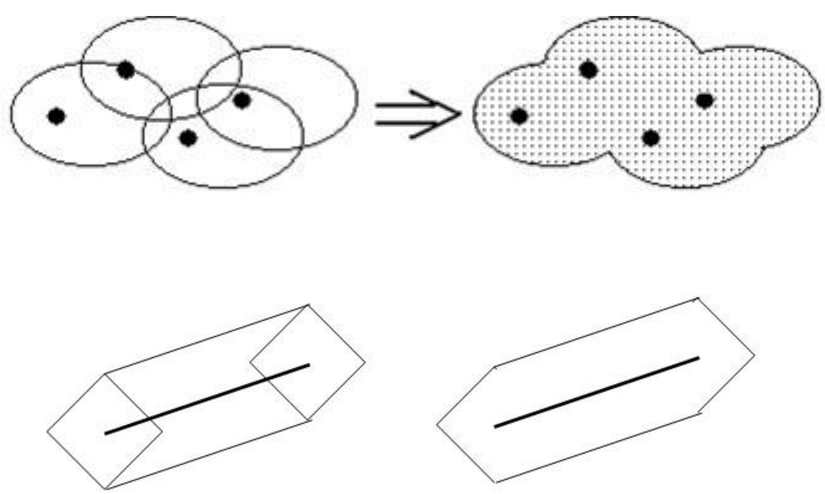


Figure 2. Examples of various dilations of a shape with various structure elements

3. Composite operations

Through combinations of dilation and erosion operations, a series of composite operations can be generated. The two well-known operations are open and close. They are defined as:

$$\text{Open operation: } S \circ B = (S \Theta B) \oplus B$$

$$\text{Close operation: } S \bullet B = (S \oplus B) \Theta B$$

Some other interesting operations may be transplanted from MM to VMM, but special algorithms must be developed to realize them. For example,a skeleton operation generates the most simplified output shape from an input shape. The algorithm is quite intuitive in MM, but is not developed in VMM.

4. Application of generalization of polygonal map

Based on the dilation and erosion operations of VMM, we designed different sequential combinations of the two operations to extract possible collision information within a single polygon and to simplify polygons. The method provides a feasible means for polygonal map generalization (e.g.a landuse map) to detect geometric conflicts and generalize polygons.

4.1 To extract possible collision information

The black polygon shown in Figure 3 is the original polygon selected from a polygonal map. We implemented inward-outward-buffering to detect collision possibly existing within the left polygon. The blue parts shown in diagram (figure 3) represent the outcome of an inward buffer and the red parts are the outcome of an outward buffer on the blue parts. The areas between the red parts cannot be identified visually on a small-scale map. That is to say, collision may happen at these areas. The performance of the inward-outward-buffering operation is analogous to the open operation of the raster-based MM. Therefore, we define the inward-outward-buffering operation is the vector-based open operation.

Besides collision, there are some small areas between red boundaries and black boundary. Someof these areas are caused by small curves of the polygon boundary. Such details may not be necessary on a smaller-scale map and may interfere the understanding of the whole features. In addition to the small curves along the boundary, some small areas are elongating along a direction but they also may not be clearly visible on a smaller-scale map. These two cases are typical geometric conflicts occurring in polygonal maps during generalization. They are typically processed in different resolutions during map generalization in MM.

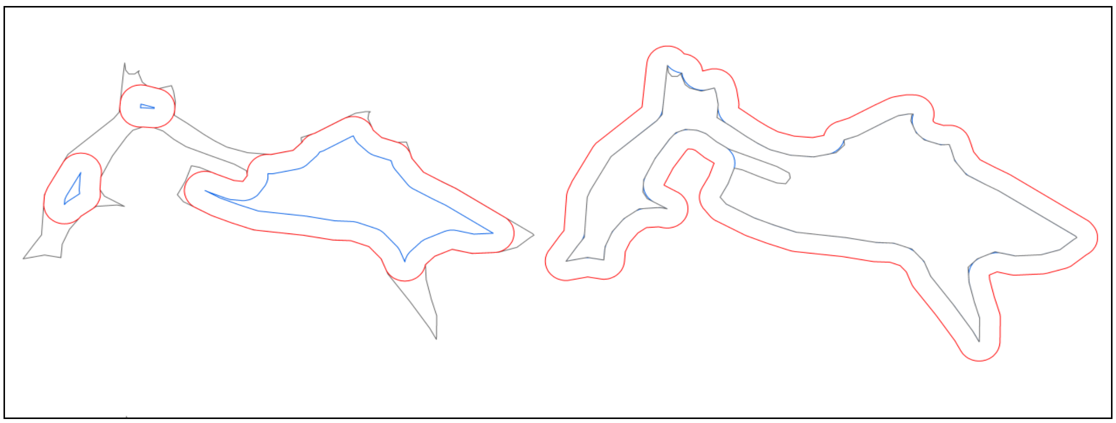
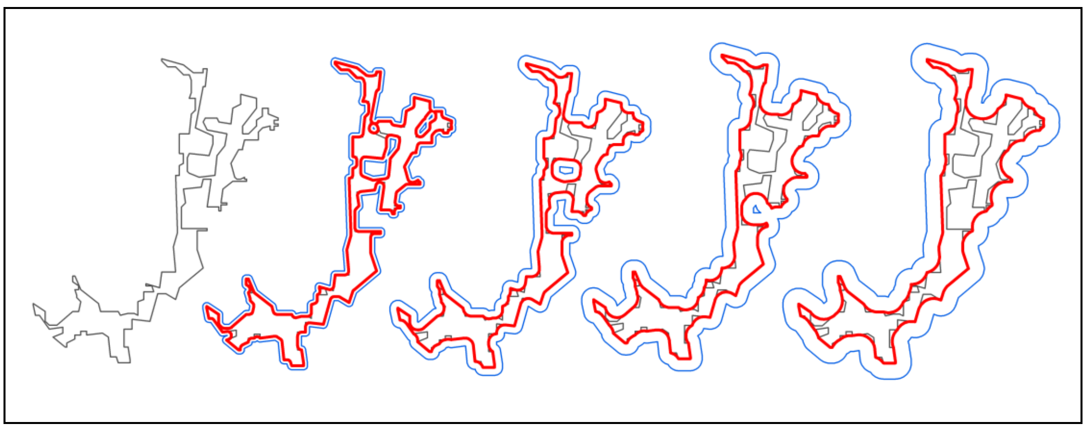


Figure 3 Performance of different sequential combination of dilation and erosion operations

4.2 To simply polygons

In the right polygon (Figure 3), outward-inward-buffering is implemented on the right polygon. The red polygon is the outcome of the outward buffering; the blue inward buffer is generated from the red polygon. The blue polygon matches closely to the black polygon except the small elongated part in the middle and along the small curves in the black boundary. The whole shape of the blue polygon is smoother and simpler than the black polygon. The performance of the outward-inward-buffering operation is analogous to the close operation of the raster-based MM. Therefore, we define the inward-outward-buffering operation as the vector-based close operation.

It is well-known that the performance of raster-based dilation and erosion operations are strongly related to the size of structure element. Likewise, the performance of vector-based operations is also impacted by the size of structure element, i.e. in this study, buffer width. For the open operation, with decreasing buffer width, fewer collision areas will exist as shown in the left polygon (Figure 4). For a close operation, with increasing buffer witdh, more details along the polygon boundary will be removed (Figure 4). The leftmost polygon in Figure 4 is the original polygon, and Figures 4 (b)-(e) are the outcomes of the close operation with the buffer width, 0.1, 0.2, 0.3, and 0.4mm respectively. The red polygons are the final polygon after the close operation. From the left to the right, the red polygons become more and more generalized. It provides an efficient solution to derive polygonal maps on different map scales.



(a) (b) (c) (d) (e)

Figure 4. Performance of the close operations with different buffer widths

5. Conclusions

Through outward and inward buffers, a Vector-based Mathematical Morphology (VMM) can be constructed in a systematic way. VMM adds some additional features to those of traditional raster-based Mathematical Morphology. As illustrated by the examples given in this paper, VMM can be applied in map generalization. Many more examples are expected to result from future studies. The work in this paper is a starting point for a new framework for processing map data.

6. Acknowledgements

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7. References

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