Comparison of Fuzzy Arithmetic and Stochastic Simulation for Uncertainty Propagation in Slope Analysis

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1. Introduction

Uncertainty is inevitable element of all data as well as processes associated with them (Zhang and Goodchild 2002). The amount and character of uncertainty is substantial for decision making that follows the process of analysing the data. To retrieve the uncertainty contained in the result of operations with data the method called propagation of uncertainty is used (Shi 2010).

In order to carry out uncertainty propagation we need theory or theories for handling uncertainty that allows such propagation. Several such theories exist including: probability theory, Dempster–Shafer theory, fuzzy sets theory, interval mathematics and others (Halpern 2003, Oberguggenberger 2005). Among those the probability theory and Stochastic Simulation (particularly represented by the method Monte Carlo) is very often used for uncertainty propagation not only in geosciences but in many other fields as well (Hanss 2005,Lodwick 2008).

2. Comparison of Fuzzy Arithmetic and Stochastic Simulation

Fuzzy arithmetic is extension of standard arithmetic operations to fuzzy numbers. Fuzzy numbers are special cases of fuzzy sets that represent uncertain or vague numbers. For the theory of fuzzy numbers and fuzzy arithmetic see Kaufmann and Gupta(1991), Hanss (2005) or Lodwick (2008). Stochastic simulation is usually represented by Monte Carlo method. For the description of the method please see Kroese (2011), Hanss (2005) or Lodwick (2008).

Oberguggenberger (2005) points out three aspects of uncertainty that need to be considered while modelling the uncertainty: definitions and axiomatics, numeric, semantics. The comparison of Fuzzy arithmetic and Stochastic simulation will be done with respect to those aspects.

Both approaches to uncertainty propagation have solid definitions, axiomatics as well as number of studies that prove their usability and correctness. The most important aspect of uncertainty is its semantics, because this is the factor that defines which theory should be used for its modelling (Hanss 2005, Oberguggenberger 2005, Shi 2010). There is no general agreement on how exactly the theory for uncertainty modelling should be chosen but several authors agree on fact that if the uncertainty comes from natural variability of data then it should be definitely modelled using statistics. But if it comes from idealization, simplification or lack of knowledge then it should be preferably modelled using fuzzy set theory (Lodwick 2008, Hanss 2005, Oberguggenberger 2005). There is also semantic issue regarding the result of the uncertainty propagation. While results of Fuzzy arithmetic can be easily interpreted as showing the possible solutions with respect to the uncertainty of inputs, the results of Stochastic simulation does not have such explicit interpretation due to the random character of the calculation (Hanss,2005; Lodwick,2008). Also different approach to the results is necessary, Fuzzy number naturally contains the uncertainty so it can be directly used for further processing. In contrast to that the results of Stochastic simulation have to be statistically processed before any further use. Oberguggenberger(2005) also points out the law of decreasing credibility which states that the stronger assumptions are used the weaker is the credibility of results. Probabilistic methods often require more information than what is available. This forces the user to use assumptions and personal opinions instead of data.

From the numeric point of view both methods provide means for uncertainty propagation. However the means and thus the results are quite different. While Stochastic simulation tells us what are the most probable outputs of the analysis, it is quite possible that the result of uncertainty propagation did not cover all the possible outcomes (Hanss 2005, Heuvelink 2002). The bigger the number of iterations of simulation the higher is the chance that more of possible outcomes will be included, however also more time and computational power will be required for such calculation and it is still practically impossible that all the possible outcomes will be included. Contrast to that Fuzzy arithmetic always covers all the possible outcomes including all the extreme solutions in the result (Hanss 2005).

Last aspect to consider is purely practical. Stochastic simulations are known to be extremely time and computational performance demanding. Both these aspects are connected with the need to generate random numbers and to store a large amount of data while performing iterations with random values (Hanss 2005, Lodwick 2008). In contrary the Fuzzy arithmetic can be calculated with significantly smaller amount of iterations than Stochastic simulation and it also does not have such high demand for storage space (Hanss 2005).

3. Case Study

Heuvelink (2002) points out that slope analysis is one the basic GIS analysis of surface. Also the results are very easy to interpret. For the two case studies presented in this paper we consider uncertain surface modelled by the field model (Burrough and McDonnell 1998). Horn’s method (sometimes also called Neighbourhood method) for calculating the slope will be used (Dunn and Hickey 1998). Heuvelink (2002) as well as other authors uses Stochastic simulation for uncertainty propagation while analysing slope of uncertain terrain. Shi(2010) and Lodwick(2008) points out that DMR is abstraction and simplification of reality. As mentioned previously it is more correct to consider such uncertainty in terms of fuzzy sets then as a random variables. The surface itself is than considered as fuzzy surface (Lodwick 2008) and Fuzzy arithmetic is used for uncertainty propagation.

Two experimental calculations were made. First case presents simple situation where the value of slope is calculated only for one cell of the grid. We assume small surface of 3×3 cell, with cell size 10 meters. We are interested in finding the possible range of result values for the middle cell if we know that the surrounding cells have value 0 meters ±1 meter (Fig. 1).

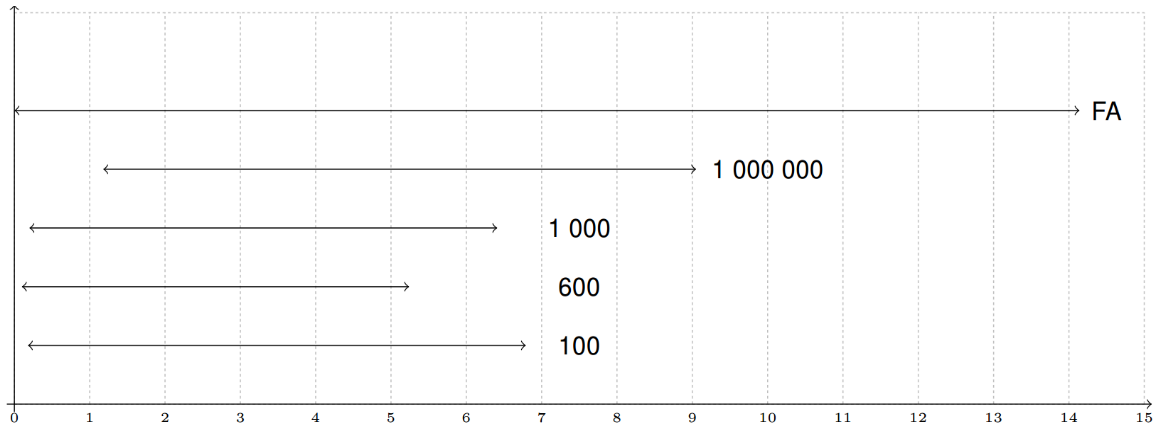

Figure 1. Representation of result interval values for several Monte Carlo realizations and Fuzzy Arithmetic

The second case presents calculation of uncertainty propagation for the grid of size 4x4 km with cell size 10x10 meters. Time demand was the main element of interest (Tab. 1). For both cases the triangular distribution was used for stochastic simulation as it is the distribution that enables using fixed upper and lower limit, also it is useful in situations when there is no exact knowledge about the distribution except for the lower and upper bound and mean value (Evans et al. 2000). Fuzzy Arithmetic is calculated for piecewise linear fuzzy numbers with 10 α-cuts, as this number of α-cuts is usually considered as good enough for most applications (Kaufmann and Gupta 1991).

Number of iterations	Time of calculation (s)
100	4.975
600	56.907
1000	91.937
Fuzzy Arithmetic	76.527

Table 1. Time necessary to perform calculation

4. Conclusion

Two methods for uncertainty propagation were compared in terms of time and memory demands as well as their ability to provide all possible solutions. Fuzzy arithmetic performed better in ability to cover all possible results. Comparison of time demands highly depends on number of iterations in case of Stochastic simulation and on number of alpha cuts in case of Fuzzy arithmetic. Slight change of those parameters can significantly affect the result of comparison. Also Fuzzy arithmetic can be further optimized by using different algorithms for calculation. Nevertheless the results of Fuzzy arithmetic offer much better foundation for further use of the results in uncertainty analysis as well as for better decision making.

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